Phase evolution in a quantum dot: Coulomb blockade versus Kondo correlation regimes

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Quantum dots

experimental device

model

discretisation of the energy levels in the dot

charge energy $e^2/2C$ ↔ Coulomb energy $U$

gate voltage $V_G$ ↔ energy $\varepsilon_0$

lateral gate voltages $V_{GL}$ and $V_{GR}$ ↔ tunnel energies $\Gamma_L$ and $\Gamma_R$

source-drain voltage ↔ $\mu_L - \mu_R$
Coulomb blockade

\[ \Gamma \ll k_B T \ll \Delta \ll U \]

- quantization of the electronic transfer through the dot
- effect of temperature

'even' valleys
'odd' valleys

state with \( N \) electrons
valleys

OFF

ON
degeneracy of the states with \( N \) and \( N+1 \) electrons
peaks
Determining the phase of a QD two path interferometer

Aharonov-Bohm oscillations of the conductance as a function of the magnetic flux $\Phi$

$$G_{AB} = \frac{2e^2}{h}|t_{ref} + t_{QD}|^2$$

$$= \text{Const} + \frac{4e^2}{h} \text{Re}(t_{ref} t_{QD}^*)$$

with $t_{ref} = |t_{ref}| e^{2i\pi\Phi/\Phi_0}$

the phase $\delta$ introduced by the QD is deduced from the shift of the oscillations with magnetic field
Experimental realization

*Two Terminal* Interferometer

*closed Aharonov-Bohm ring*

\[ t_{SD}(B) = t_{DS}(-B) \]

phase rigidity ........... \( G(B) = G(-B) \)

*Four Terminal* Interferometer

*two path* interferometer

\[ t_{Sb Db}(B) = t_{bD bs}(-B) \]

no phase rigidity ........ \( I_D(B) \neq I_D(-B) \)
two terminal

Yacoby et al PRL 74, 4047 (95)

four terminal

Schuster et al Nature 385, 417 (97)
Due to time reversal symmetry, $G(B) = G(-B)$, the phase can equal either 0 or $\pi$. Argument of phase rigidity.

Levy Yeyati, Büttiker PRB 52, R14360 (95)
Phase Evolution in a *Four Terminal Interferometer*

Two path interferometer

Collector Voltage (a.u.)

Plunger Gate Voltage (V)

\[ \Delta \theta (t_Q) \]

Collector Voltage (a.u.)

Magnetic Field (mT)
Phase Evolution of few CB Peaks in the *CB Regime*

All the peaks have the same phase: gradual increase from 0 to $\pi$ followed by an abrupt phase lapse
Schuster et al interpreted their results obtained in the CB regime in terms of resonant transmission.

**One resonance**

Breit-Wigner formula for the transmission amplitude:

\[ t = C_n \frac{1}{E - E_n + i\Gamma} \]

\[ \arg(t) = \arg(C_n) + \arctg \frac{\Gamma}{E_n - E} \]

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Succession of resonances

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[Graph showing the behavior of \(|t|^2\) and \(\arg(t)\) with energy, and a graph showing the succession of resonances with \(|t|^2\) and \(\arg(t)/\pi\) versus \(W\).]
• Why the abrupt phase lapse?
• Why all the resonances with the same phase?

No complete explanation yet!

We will come back to these questions at the end of the talk
Kondo effect

\[ k_B T < T_K < \Gamma < \Delta < U \]

\[ T_K = (\Gamma U)^{1/2} \exp[\pi \varepsilon_0 (\varepsilon_0 + U)/\Gamma U] \]

Due to the Kondo effect, the conductance in the odd valleys increases with decreasing temperatures until reaching the unitary limit.

Goldhaber-Gordon PRL'99 and Nature'98
Kouwenhoven Physics World'98

van der Wiel, De Franceschi et al Science 289, 2105 (2000)
The Kondo effect in bulk metals

Increase of the resistivity with decreasing temperatures

metal impurity
spin-flip
Kondo resonance
virtual bound state
electronic correlations
The Kondo effect in quantum dots

The spin-flip process increases the conductivity in the ‘odd’ valleys at low temperatures.
Recovering the Coulomb blockade regime by quenching the Kondo effect

Heiblum, transparencies at the KITP
Santa Barbara Web site (2001)
Determination of the phase and visibility of the Kondo QD using an AB interferometer
Evolution of the phase when reducing coupling strength

Unitary limit

Uncomplete phase lapse

Weak correlations

Coulomb blockade

plateau

Ji, Heiblum et Shtrikman PRL 88, 076601 (02)
So far there has been

• no interpretation of the exp. results for the phase
• no interpretation of the phase lapses

For the Kondo effect in bulk metals

\[ \rho \propto \sin^2 \delta \Rightarrow \delta = \pi / 2 \]

Langreth PR 150, 516 (66) and Nozières JLTP 17, 31 (74)

For the Kondo effect in QD

NRG and Bethe-ansatz calculations \[ \delta = \pi/2 \]

Gerland et al PRL 84, 3710 (2000)
Two reservoir Anderson model

Glazman et Raikh JETP Lett. 47, 452 (88)
Ng et Lee PRL 61, 1768 (88)

\[ H = \sum_{k\sigma; i=L,R} \varepsilon_k c_{k\sigma i}^{\dagger} c_{k\sigma i} + \varepsilon_0 \sum_{\sigma} n_{d\sigma} + \sum_{k\sigma; i=L,R} V_i (d_{\sigma i}^{\dagger} c_{k\sigma i} + h.c.) + U n_{d\uparrow} n_{d\downarrow} \]
One reservoir Anderson model

\[ H = \sum_{k\sigma} \varepsilon_k \alpha_{k\sigma}^\dagger \alpha_{k\sigma} + \sum_{k\sigma} \varepsilon_k \beta_{k\sigma}^\dagger \beta_{k\sigma} + \varepsilon_0 \sum_{\sigma} n_{d\sigma} + U n_{d\uparrow} n_{d\downarrow} + \sum_{k\sigma} \tilde{V} (d_{\sigma}^\dagger \alpha_{k\sigma} + h.c.) \]

with

\[ \tilde{V} = \sqrt{|V_L|^2 + |V_R|^2} \]

and

\[
\begin{cases}
\alpha_{k\sigma} = (V_L c_{k\sigma L} + V_R c_{k\sigma R}) / \tilde{V} \\
\beta_{k\sigma} = (-V_R c_{k\sigma L} + V_L c_{k\sigma R}) / \tilde{V}
\end{cases}
\]
Scattering theory at 1D

\[
\begin{align*}
\psi_{k\sigma L} &= u_{k\sigma L} \exp^{ikx} + v_{k\sigma L} \exp^{-ikx} \\
\psi_{k\sigma R} &= u_{k\sigma R} \exp^{ikx} + v_{k\sigma R} \exp^{-ikx}
\end{align*}
\]

\[
\begin{pmatrix}
  u_{k\sigma L} \\
u_{k\sigma R}
\end{pmatrix} = C_\sigma 
\begin{pmatrix}
  1 - it_{k\sigma} & -it_{k\sigma} \\
  -it_{k\sigma} & 1 + it_{k\sigma}
\end{pmatrix} 
\begin{pmatrix}
  v_{k\sigma L} \\
u_{k\sigma R}
\end{pmatrix}
\]

with \( t_{k\sigma} = 2\pi V^2 N_\sigma(\varepsilon_k) G_\sigma(\varepsilon_k + i\eta) \)
In the absence of magnetic moment (for instance in the Kondo regime at $T=0$)

$$S_{k\sigma} = C_\sigma \exp^{i\delta_\sigma} \begin{pmatrix} \cos \delta_\sigma & i \sin \delta_\sigma \\ i \sin \delta_\sigma & \cos \delta_\sigma \end{pmatrix}$$

with $\delta_\sigma = \pi n_{d\sigma}$ Friedel sum rule

Using the extended Levinson theorem

$$\ln \det S_\sigma = 2i\pi n_d \quad \text{hence} \quad C_\sigma = \exp^{i\delta_{-\sigma}}$$

A. Jerez, P. Vitushinsky, M.L. PRL 95, 127203 (2005)
The transmission amplitude is given by
\[ t_{QD} \propto i \exp(i\delta) \sin\delta_{\sigma} \] with \( \delta = \delta_\uparrow + \delta_\downarrow = \pi n_d \)

Using the Landauer formula
\[ G \propto T = \sum_{\sigma} |t_{QD}|^2 = \sum_{\sigma} \sin^2 \delta_{\sigma} \]
\[ G \propto 2 \sin^2(\delta/2) \]

Aharonov-Bohm interferometry

\[ G_{AB} = \frac{2e^2}{h} |t_{\text{ref}} + t_{QD}|^2 = \text{Const} + \frac{4e^2}{h} \text{Re}(t_{\text{ref}}t_{QD}^*) \]

A. Jerez, P. Vitushinsky, M.L. PRL '05
Check of the prediction

\[ G \propto 2 \sin^2(\delta/2) \]

The Anderson model is integrable
We have numerically solved the Bethe ansatz equations to derive $n_d$ and hence $\delta/\pi$

We have derived benefit from the universal behavior as a function of a renormalized energy $\varepsilon_0^*$

\[ \varepsilon_0^*/\Gamma = \varepsilon^*/\Gamma + \beta \log(\alpha \Gamma / U) \]
Fit in the unitary limit and Kondo regimes
Fit of the experimental data in the presence of a single fitting parameter $\Gamma/U$

(a) Unitary limit

(taking $V_G/\varepsilon_0=30\text{mV}$)

(b) Weak correlation

A.Jerez, P. Vitushinsky, M.L.
PRL '05
Interpretation of the uncomplete phase lapse in Kondo QD

In the non magnetic regime (for instance Kondo regime at T=0)

\[ t_{QD} = i e^{i\delta} \sin(\delta/2) \]

\[ I_{osc} \propto \sin(\delta/2) \sin(\theta - \delta) \]

with \( \theta = 2\pi \Phi/\Phi_0 \)

Phase lapse of value \( \pi \) (incomplete)

P. Vitushinsky, M.L., A.Jerez (in preparation)
Interpretation of the complete phase lapse in the Coulomb blockade regime

\[ I_{osc} \propto \text{Re}(t_{ref} t_{QD}^*) \]

For the Breit-Wigner resonance (Coulomb blockade regime)

\[ t_{QD} = i e^{i\delta} \sin \delta \]

\[ I_{osc} \propto \sin \delta \sin(\theta - \delta) \]

Phase lapse of value \( \pi \) (complete)
Conclusions

\[ G \propto \sum_\sigma \sin^2(\delta_{QD}/2) \]

\[ \delta_{QD} = \pi n_d \]
Non magnetic regime

\[ t_{QD} = i e^{i\delta} \sin(\delta/2) \]

Phase lapse of value \( \pi \) (incomplete)

CB regime

\[ t_{QD} = i e^{i\delta} \sin \delta \]

Phase lapse of value \( \pi \) (complete)