Measuring the current-phase relation in superconducting atomic contacts

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Constitutive relations for normal metal discrete elements

\[ V = R \frac{dV}{dt} \]
\[ V = L \frac{dI}{dt} \]
\[ C \frac{dV}{dt} = I \]

impedance \( Z(\omega) = \frac{V(\omega)}{I(\omega)} \)

\( Z(\omega) = R \)

\( Z(\omega) = \frac{1}{jC\omega} \)

\( Z(\omega) = jL\omega \)
Constitutive relations for superconducting discrete elements

\[ V = \frac{\hbar}{2e} \frac{d\delta}{dt} \]

- **Josephson Junction**
  \[ I(\delta) = I_0 \sin(\delta) \]

- **SFS junction**
  \[ I(\delta) = I_0 \sin(\delta + \pi) \]

- **Point contact**
  \[ I(\delta) = \sum_{\{\tau_i\}} \frac{e\Delta}{2\hbar} \tau_i \frac{\sin(\delta)}{\sqrt{1 - \tau_i \sin^2(\delta/2)}} \]

Methods to measure \( I(\delta) \)?
Strategies to measure $I(\delta)$

<table>
<thead>
<tr>
<th>Imose (bias)</th>
<th>Measure</th>
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<tr>
<td>$\delta$</td>
<td>$\delta$ or resulting $B$</td>
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<td>$\dot{\delta}$</td>
<td>Shapiro steps</td>
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<tr>
<td>$\delta$</td>
<td>$\frac{\partial I}{\partial \delta}$</td>
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</table>
Phase biasing

Fluxoid quantization: \( \delta + 2\pi \frac{\Phi}{\Phi_0} = n.2\pi \)

( or \( \delta + \varphi = n.2\pi \) with \( \varphi = 2\pi \frac{\Phi}{\Phi_0} \) )

Flux in the loop: \( \Phi = \Phi_{\text{ext}} + \Phi_{\text{in}} \)

\( \Phi_{\text{in}} = LI(\delta) \)

\( \varphi + \frac{LI(\varphi)}{\varphi_0} = \varphi_{\text{ext}} \)

(\( \varphi_0 = \frac{\hbar}{2e} \))
Phase biasing

\[ \varphi + \frac{L I(\varphi)}{\varphi_0} = \varphi_{\text{ext}} \]

\[ L \left| \frac{\partial I(\varphi)}{\partial \varphi} \right|_{\text{Min}} < \varphi_0 \ ? \]

Measure \( \varphi(\varphi_{\text{ext}}) \)

\[ \Rightarrow I(\varphi) \]

Only part of the curve is accessible!

Small signal!
Phase biasing: measurement methods

\[ \varphi_{\text{ext}} = \varphi + \frac{L I(\varphi)}{\varphi_0} \]

Measure \( \varphi(\varphi_{\text{ext}}) \) \( \Rightarrow I(\varphi) \)

1. Place on a Hall bar and measure \( V_{\text{Hall}} \)
2. Couple to the pick-up coil of a SQUID
1. Measurement of $I(\delta)$ a Pi-junction with a Hall-bar

(Bauer et al., PRL 92, 217001 (2004))
2. Measurement of $I(\delta)$ a break junction with a SQUID

(Koops et al., PRL 77, 2542 (1996))

\[ I(\delta) = \sum_{\{\tau_i\}} \frac{e\Delta}{2\hbar} \frac{\sin(\delta)}{\sqrt{1 - \tau_i \sin^2(\delta/2)}} \]

Non-sinusoidal, but what was the sample?
Supercurrent through a ballistic single channel ($\tau = 1$)

Andreev spectrum

$$E_\pm = \pm \Delta \cos\left(\frac{\delta}{2}\right)$$

Current-phase relationship

$$I_\pm = \frac{2e}{\hbar} \frac{\partial E_\pm}{\partial \delta}$$
Supercurrent through a non-ballistic single channel \((\tau < 1)\)

\[
\delta = \phi_L - \phi_R \quad \text{(phase bias)}
\]

Andreev spectrum

\[
E(\delta) = \pm \Delta \sqrt{1 - \tau} \quad \pi \quad 2\pi 
\]

Current-phase relationship

\[
l_\pm(\delta) = \mp \frac{e\Delta}{2\hbar} \tau \frac{\sin(\delta)}{\sqrt{1 - \tau \sin^2 \left(\frac{\delta}{2}\right)}}
\]

Bagwell, PRB 46, 12573 (1992)
Determination of the channels content of an atomic point contact

- $V_J / (\Delta / e)$
- $I / I_c$
- $\Delta = 178 \mu V$
- $I_c = 22.9 \text{nA}$

DC Josephson supercurrent peak

Multiple Andreev Reflections

Counter-support

Insulating layer

Pushing rod
I-V of a one-channel contact

Averin & Bardas, PRL 75, 1831 (1995)
Cuevas et al., PRB 54, 7366 (1996)
V. Shumeiko et al., LTP 23, 181 (1997)
Multiple Andreev Reflexions

Blonder, Tinkham, Klapwijk (‘82)
Channels decomposition of the I-V of an atomic contact

Fingerprint of mesoscopic PIN code

\[ I_{\text{exp}} = \sum_{n} I_{0}(V, \tau_{n}) \]

I-V characteristic

\[ I / (G_{0} \Delta) \]

\[ eV / \Delta \]

\[ \{\tau_{1}, ..., \tau_{N}\} \]
Measurement of the current-phase relation in atomic point contacts

\[ I(\delta) = \sum_{\{\tau_i\}} \frac{e\Delta}{2\hbar} \tau_i \sin(\delta) \sqrt{1 - \tau_i \sin^2(\delta/2)} \]

- Measure \( I(V) \) to get the channels transmissions
- Determine \( I(\delta) \)

Both voltage and phase bias required!

connected sample
Strategies to measure $I(\delta)$

**Impose (bias)**

$\delta$

**Measure**

$\dot{\delta}$

or resulting $B$

$\delta$

??

Shapiro steps

$\delta$

Microwave measurements

$\partial I / \partial \delta$
Voltage biasing: ac Josephson effect

\[ \delta(t) = \frac{2eV}{\hbar} t = \Omega_J t \]

\[ \dot{\delta} \text{ biasing} \]

ac Josephson currents

\[ I(\omega) \]

Detection?
Shapiro steps and $I(\delta)$

\[ I(\delta) = A \cos(\omega_r t) \]

\[ \delta(t) = \delta_0 + \frac{V}{\phi_0} t + 2\alpha \sin(\omega_r t) \]

with \( V = \frac{1}{2} \phi_0 \omega_r \) and \( \delta_0 \) maximizing \( \langle I(t) \rangle \)

Phase bias

\[ I(\delta) / I_0 \]

\[ \delta / (2\pi) \]

\( \tau = 0.995 \)

Voltage bias

\[ \langle I(t) \rangle \neq 0 \]

If \( q\omega_0 = p\omega_r \)

\( \omega_r t \)
Shapiro steps and $I(\delta)$

\[ I(\delta) = A \cos(\omega_r t) \]

\[ \delta(t) = \delta_0 + \frac{V}{\varphi_0} t + 2\alpha \sin(\omega_r t) \]

with

\[
\begin{align*}
V &= \frac{1}{2} \varphi_0 \omega_r \\
\delta_0 &= \text{maximizing } \langle I(t) \rangle
\end{align*}
\]

ac + dc voltage bias

\[
\langle I(t) \rangle \neq 0
\]

If $q\omega_0 = p\omega_r$
Measurement setup

Voltage bias of the Josephson junction
Measurement of Shapiro steps

\[ \{0.5\} \]

\[ \{0.988, 0.321, 0.102, 0.061\} \]

\[ eV = \frac{1}{2} \times \hbar \omega_r \]

\[ eV = 1 \times \hbar \omega_r \]
Why are fractional Shapiro steps so small?

Dynamics of $\delta(t)$ near Shapiro step at $eV = \frac{p}{q} \times \hbar \omega_r$ obeys the same Langevin equation as supercurrent with:

$$V_{\text{eff}} = V - \frac{p \hbar \nu}{q 2e}$$

$$I_{\text{c}}^{\text{eff}} = I_q \cdot J_p(2q\alpha)$$

$$T^{\text{eff}} = q \cdot T$$

It is difficult to observe high order harmonics.
## Strategies to measure $I(\delta)$

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<td>$\delta$</td>
<td>$\delta$ with an additional Josephson Junction</td>
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<tr>
<td>$\dot{\delta}$</td>
<td>$\dot{\delta}$ Shapiro steps</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$\frac{\partial I}{\partial \delta}$ Microwave measurements</td>
</tr>
</tbody>
</table>
Allowing both for phase biasing AND I-V measurements

Phase bias Measure $I$ (?)

I-V measurements

$V$ (µV)

$I$ (µA)

I $\rightarrow$ OPEN

SHORT
1. Measurement the I-V of the atomic contact

Parameters: \( I_c = 780 \text{ nA} \)

\[ i_c = \frac{e\Delta}{\hbar} \approx 50 \text{ nA for } \tau = 1 \]
1. Measurement the I-V of the atomic contact

1) Measure only

2) Measure and together

3) Subtract

\[
\Delta = 178 \, \mu eV \\
\tau = \{0.475 ; 0.138 ; 0.019\}
\]

\[
V = \frac{2\Delta}{e}
\]

\[\{\tau_1, \ldots, \tau_N\}\]
2. Measurement of $I(\delta)$

**METHOD 1**: “switching”

- Measure $I_S(\Phi)$

**METHOD 2**: microwave reflectometry

Measure $Z = (L(\Phi)/C)^{1/2}$
From $I(\delta)$ to the switching current
From $I(\delta)$ to the switching current

\[ I = i_c f(\delta) + I_c \sin \gamma \]

\[ \delta - \gamma = \varphi \]

Approximation: $i_c \ll I_c$

\[ \Rightarrow \text{switching near } \gamma = \pi / 2 \]

\[ \Rightarrow I_{sw} \approx i_c f(\varphi + \pi / 2) + I_c \]

\[ \tau = \{0.475, 0.138, 0.019\} \]

\[ \tau = \{0.983\} \]
From $I(\delta)$ to the switching current

$I = i_c f(\delta) + I_c \sin \gamma$

$\delta - \gamma = \varphi$

Approximation: $i_c \ll I_c$

$\Rightarrow$ switching near $\gamma = \pi / 2$

$\Rightarrow I_{sw} \approx i_c f(\varphi + \pi / 2) + I_c$

$\tau = \{0.475, 0.138, 0.019\}$

$\tau = \{0.983\}$
From $I(\delta)$ to the switching current

$\delta \rightarrow \phi \bigotimes \gamma \rightarrow i \rightarrow I_J$

Comparison with

$I(\delta) = \sum_{\{\tau\}} \frac{e\Delta}{2\hbar} \tau_i \frac{\sin(\delta)}{\sqrt{1 - \tau_i \sin^2 (\delta / 2)}}$

assuming $\delta = \varphi + \frac{\pi}{2}$ and $\varphi = \varphi_{\text{ext}}$

$\tau = \{0.475, 0.138, 0.019\}$

$\tau = \{0.983\}$

$T_{\text{fridge}} = 26 \text{ mK}$

$\Delta = 178 \text{ µeV}$
Measurement of $L(\delta)$: microwave reflectometry

$$L(\delta) = \frac{\varphi_0}{dI(\delta)/d\delta}$$

$$L_{tot}^{-1}(\varphi) = L_{tot}^{-1}(\delta) + L_{j}^{-1}(\gamma)$$

$$\delta = \varphi + \arcsin \left( \frac{l - i_c f(\delta)}{l_c} \right)$$

Two knobs
microwave reflectometry: experimental setup

\[ U = U_0 \cos(2\pi vt) \]

\[ R = 50 \, \Omega \]

\[ U_i = U/2 \]

\[ C \approx 35 \, \text{pF} \]

\[ I_c = 780 \, \text{nA} \]

\[ \delta \]

\[ \phi \]

\[ \gamma \]

\[ Z = \sqrt{\frac{L_{tot}(\varphi, I)}{C}} \]

\[ R_v \approx \frac{Z - Z_0}{Z + Z_0} \]
microwave reflectometry : changing $R$ with $\phi$ or $I$

\[
\tau = \{0.475, 0.138, 0.019\}
\]

\[
\nu = 1.24 \text{ GHz} \quad \text{C} = 33.7 \text{ pF}
\]

\[
I_t^0 = 750 \text{ nA} \quad r = 0.57 \Omega
\]

\[
Z_0 = 50 \Omega \quad \Delta = 178 \mu \text{eV}
\]

\[
L_{tot}^{-1}(\varphi, I) = L_{\delta}^{-1}(\varphi) + L_J^{-1}(\gamma)
\]

\[
L_J^{-1}(\gamma) = \frac{I_c}{\varphi_0} \cos \gamma \approx \frac{1}{\varphi_0} \sqrt{I_c^2 - I^2}
\]

\[
L_{tot}^{-1}(\varphi, I) \approx L_{\varphi}^{-1}(\varphi + \arcsin \frac{I}{I_c}) + L_J^{-1}(I)
\]

Low transmissions

\[
|R_n| (\text{dB})
\]

\[
\frac{I}{I_c^0}
\]

(\text{also data on Arg}(R_v))
microwave reflectometry: changing $R$ with $\phi$ or $I$

Large transmission

$\tau = \{0.983\}$

$\nu = 1.24 \ \text{GHz}$
$I_r^0 = 750 \ \text{nA}$
$C = 33.7 \ \text{pF}$
$r = 0.57 \ \Omega$
$Z_0 = 50 \ \Omega$
$\Delta = 178 \ \mu\text{eV}$
microwave reflectometry : effect of \( T \)

\[ \tau = \{0.983\} \]

Effect of quantum fluctuations of \( \delta \)?
CONCLUSIONS

\[ \Delta = 178.4 \mu eV \]
\[ \tau = 0.988, 0.321, 0.102, 0.061 \]
\[ V_c = 10.06 \text{ GHz} \]
\[ T = 180 \text{ mK} \]
\[ D = 178.4 \mu eV \]
\[ \tau = 0.983 \]
\[ T_{\text{fridge}} = 26 \text{ mK} \]
\[ |R_n| (\text{dB}) \]