Origin of hysteresis in a proximity Josephson junction

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Outline

The S-N-S junction
Electron thermometry with N-I-S junctions
The origin of hysteresis in S-N-S junctions
How do electrons thermalize?
The S-N-S junction
A diffusive S-N-S Josephson junction

Elastic m.f.p. \( l_e \approx 10 \text{ nm} \)

Thermal length \( L_T = \sqrt{\frac{\hbar D}{2\pi k_B T}} \)
\[ \approx 0.1 \text{ \mu m} \text{ at 1 K} \]

Sample length \( L \approx 1 \text{ \mu m} \)

Phase-breaking \( L_{\varphi} \approx 3 \text{ \mu m} \)
The Josephson supercurrent

In long junctions $L \gg \xi_S$

Energy scale = Thouless energy:

$$\varepsilon_c = \frac{\hbar D}{L^2}$$

The critical current:

$$eR_N I_C = \frac{32}{3 + 2\sqrt{2}} \varepsilon_c \left[ \frac{L}{L_T} \right]^{3/2} \exp\left(-\frac{L}{L_T}\right)$$

At non-zero voltage: the R(C)SJ model

Resistively and Capacitively Shunted Junction:

\[ I = \frac{V}{R_n} + C \frac{dV}{dt} + I_c \sin \varphi \]

and:

\[ \frac{d \varphi}{dt} = \frac{2eV}{\hbar} \]

Combining the two gives:

\[ \frac{d^2 \varphi}{d\tau^2} + Q^{-1} \frac{d \varphi}{d\tau} + \sin \varphi = \frac{I}{I_c} \]

with \( \omega_p = \sqrt{\frac{2eI_c}{\hbar C}} \), \( \tau = \omega_p t \),

\[ Q = \omega_p R_n C \quad \text{(quality factor)} \]
The R(C)SJ model

Equivalent to a ball in a tilted washboard.
position = $\varphi$ ← current
velocity = voltage

$Q > 1$ (tunnel junctions): full RSCJ model.
Underdamped regime:
after escape, kinetic energy is conserved,
hysteretic I(V).

$Q < 1$ (lateral junctions): RSJ model.
Overdamped regime: zero mass,
non-hysteretic I(V) predicted.

\[ V = R_n \sqrt{|I|^2 - |I_c|^2} \quad (T = 0) \]
Hysteresis in S-N-S junctions, ...

Routinely observed when critical current is large. Possibly thermal origin, hypothesis of an intrinsic effect (effective capacitance).

From Nb nanowires templated by a CNT to InAs semiconducting nanowires...

... and carbon-based devices.

From carbon nanotubes to graphene ...
Electron thermometry with N-I-S junctions
Threshold in the $I(V)$ smeared by the (electronic) temperature in N:

\[ I_T = \frac{1}{eR_n} \int_{-\infty}^{+\infty} n_S(E) \left[ f_S(E - eV) - f_N(E) \right] dE \]

At a fixed bias current, the voltage measures temperature.
The origin of hysteresis in S-N-S junctions
Our samples

$L = 1.5 \, \mu m$, $w = 0.17 \, \mu m$, $t = 30 \, nm$

$R_n = 10 \, \Omega$ : $D = 100 \, cm^2/s$, $l_e = 22 \, nm$

Thouless energy $\varepsilon_c = 28 \, mK = 2.4 \, \mu eV$
The thermometer calibration

I(V) of the double N-I-S junction at thermal equilibrium.

Calibration at a fixed current bias: no saturation down to 40 mK.

Hypothesis of quasi-equilibrium in the N metal: $T_e$ can be defined.
I(V) and electron temperature

The electron thermometer correlates to switching.

Thermal origin of the hysteresis.

$T_e$ up to 0.6 K!
The thermal instability

Dissipation heats above $T_{\text{bath}}$, which reduces $I_c$, which increases dissipation, ...
The critical current

Good fit to theory.

Retrapping current close to equilibrium value at $T_e$.

Quasi-equilibrium situation.

<table>
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<tr>
<th>#</th>
<th>L (µm)</th>
<th>$R_n$ (Ω)</th>
<th>W (nm)</th>
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How do electrons thermalize?
A basic model for the heat balance

Joule power is deposited in the e- population: \( P_J = I \cdot V \)

Electron-phonon interaction in N:

Heat balance: \( P_J = P_{e-ph} \) and \( T_{ph} \) small,

\[
T_e = \sqrt[5]{\frac{IV}{\Sigma U}}
\]

Gives \( T_e \) of about 1 K: too large.
Thermal flow through the S electrodes

Thermal conductivity of a metal (WF law):

\[ g_N^T = L_0 g_N T \]

In the S state, scaled by:

\[ r(T) = \frac{3}{2\pi^2} \int_{\Delta/k_B T}^{\infty} \left( \frac{x}{\cosh(x/2)} \right)^2 dx \]

Power flow:

\[ P_K = L_0 G_N^{sc} \int_{T_e}^{T_b} r(T) T dT \]
Good fit with the S electrodes thermal conductance.

e-ph coupling not enough to fit data.
Conclusion

Thermal origin of the hysteresis in long S-N-S junctions.

Should hold in shorter junctions, junctions based on nanowires, carbon nanotubes, graphene ...

Power density at switching about 1 nW/µm³ while here $2 \times 10^{-3}$ - 1 nW/µm³.

Heat link is here dominated by the S electrodes.