Physical consequences of electron-electron interactions in graphene Landau levels

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Outline

• The role of electronic interactions in graphene (introduction)

• Collective particle-hole excitations in the IQHE regime

• Bernstein modes in graphene

• Strong correlations and the FQHE in graphene

Dirac fermions in graphene

- Undoped graphene: lattice half-filled with electrons

$\Rightarrow$ Zero-energy states ($\varepsilon_{\mathbf{K}\pm} = 0$):

- at $K$ and $K'$ points of the 1st BZ

- Continuum limit $\mathbf{k} = \mathbf{K} \pm \mathbf{q}$ with $|\mathbf{q}| \ll 1/a$:

$$\mathcal{H}^{\xi=\pm}(\mathbf{q}) = \xi \frac{3}{2} t a \begin{pmatrix} 0 & q_x - i q_y \\ q_x + i q_y & 0 \end{pmatrix} = \xi \hbar v_F \sigma \cdot \mathbf{q}$$

2D Dirac Hamiltonian for massless particles

- Energy dispersion (two-fold valley degeneracy $\xi = \pm$):

$$\varepsilon_{\lambda=\pm, \mathbf{q}}^{\xi=\pm} = \pm \hbar v_F |\mathbf{q}|$$
Interactions in graphene – coupling constant & screening

Coulomb interaction in 2DEG

\[ r_s = \frac{e^2}{\frac{eL}{\hbar^2} \frac{2m_B L^2}{2m_B L^2}} = \frac{1}{a_0 k_F} \propto n_{el}^{-1/2} \]

⇒ electron correlations relevant at low densities

... and in graphene

\[ \alpha_G = \frac{e^2}{\frac{eL}{\hbar v_F} L} = \frac{e^2}{\hbar \epsilon v_F} \approx \frac{2}{\epsilon} \]

⇒ density-independent (fine-structure constant)
**Interactions in graphene – coupling constant & screening**

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Interaction range in 2DEG

\[ k_{TF} = r_s k_F = a_0^{-1} \]

⇒ density-independent screening length \( \lambda_{TF} \sim a_0 \)

... and in graphene

\[ k_{TF} \sim 2k_F/\epsilon \sim \sqrt{n_{el}} \]

⇒ no TF screening at Dirac points, \( \lambda_{TF} \gg a \)
Graphene in a Magnetic Field

• Magnetic field \( (B\mathbf{e}_z = \nabla \times \mathbf{A}) \) via Peierls substitution:

\[
\hbar \kappa \rightarrow -i\hbar \nabla + e\mathbf{A}
\]

(semiclassical: \( \varepsilon(\kappa) \rightarrow \varepsilon(\sqrt{2n/l_B}), \ l_B = \sqrt{\hbar/eB} \))

• Energy dispersion with magnetic field (degenerate in valley isospin \( \xi \)):

\[
\varepsilon_{\lambda,n} = \lambda \hbar \frac{v_F}{l_B} \sqrt{2|n|} \propto \sqrt{B|n|}
\]

(Relativistic LLs)

• Quantum Hall effect at

\[\nu = \frac{n_{el}}{eB/\hbar} = \pm 2, \pm 6, \pm 10, \ldots\]
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**IQHE in graphene**


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**Graphene IQHE:**

\[ R_H = \frac{h}{e^2} \nu \]

at \( \nu = 2(2n+1) \)

**Usual IQHE:**

at \( \nu = 2n \)

(no Zeeman)

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Density of states

\( V_g = 15V \)

\( T = 30 \text{mK} \)

\( B = 9T \)

\( T = 1.6K \)

\( \sim \nu \)

\( \sim 1/\nu \)
Correlations in a strong magnetic field

Integer filling factors

Partial filling factors

“weak” correlations

→ perturbation expansion in

\[ \alpha_G = \frac{e^2}{\epsilon R_C} / \Delta_n \sim \frac{e^2}{\hbar \epsilon v_F} \]

→ collective PH excitations

“strong” correlations

→ kinetic energy irrelevant (quenched)

→ realm of the FQHE
Collective Excitations in Doped Graphene in the IQHE regime

in collaboration with:

R. Roldán (University of Nijmegen)

and J.-N. Fuchs (LPS)


arXiv: 1012.0779
Particle-hole excitations in graphene

Band dispersion: \( \varepsilon_{\lambda=\pm,q} = \lambda \hbar v_F |q| \)

Polarisation diagram ("electron-hole bubble"): 

\[
\Pi^0(q, \omega) = \sum_{\lambda, \lambda', k} \frac{1 + \lambda \lambda' \cos(\theta_{k,q})}{2} \frac{n(\varepsilon_{\lambda', q+k}) - n(\varepsilon_{\lambda, k})}{\omega - (\varepsilon_{\lambda, k} - \varepsilon_{\lambda', q+k}) + i\delta}
\]

absence of backscattering \( \rightarrow \) concentration around \( \omega = \hbar v_F |q| \)
**SPES of graphene in a magnetic field**

Landau levels (LLs): 
\[
\varepsilon_{\lambda,n} = \lambda \frac{\hbar v_F}{l_B} \sqrt{2n}
\]

- LLs no longer equidistant 
  \( [\Pi^0(\omega, q) = \text{long & boring formula}] \)
  \( \Rightarrow \) no horizontal magneto-excitons
  \( \Rightarrow \) weight around diagonal lines ("linear magneto-plasmons")
Particle-hole excitations in the RPA

Electron-hole bubble diagram in the RPA:

\[ \Pi^{RPA}(q, \omega) = \Pi^0(q, \omega) / \left[ 1 - v(q) \Pi^0(q, \omega) \right] \]

- appearance of 2D plasmon mode with \( \omega_{\text{plas}} \sim \sqrt{q} \)
- magneto-excitons (2DEG) acquire weak dispersion
- linear magneto-plasmons (graphene) more pronounced
Bernstein modes in high-mobility graphene

Hybridisation of plasmon mode with inter-LL transitions

Dyson-type equation (at resonance):

$$\omega^2 \pm = \frac{\omega^{\text{plas}}(q)^2 + \Omega^{2}_{\lambda n, n'}}{2} \pm \sqrt{\frac{[\omega^{\text{plas}}(q)^2 - \Omega^{2}_{\lambda n, n'}]^2}{4} + 4 \left( \frac{e^2}{eql^2 B} \right)^2 \omega^{\text{plas}} \Omega_{\lambda n, n'} |F_{\lambda n, n'}(q)|^2}$$
SU(4) Fractional Quantum Hall Effect in Graphene

in collaboration with :

N. Regnault (LPA-ENS), R. de Gail (LPS),

and Z. Papić (now: Princeton University)

FQHE – Laughlin Wave Function

Partially filled LL

→ interactions relevant

Perturbation theory fails!

Lowest-LL wave functions:

• analytic functions:

\[ \psi_m^{(1)}(z) = z^m \exp(-|z|^2/4) \]

• translation/rotation invariance for 2 fermions:

\[ \psi^{(2)}(z, z') \propto (z - z')^{2s+1} \]

Laughlin's one-component wave function (1983):

\[ \psi^L_s(\{z_j\}) = \prod_{i<j} (z_i - z_j)^{2s+1} e^{-\sum_k |z_k|^2/4} \quad \text{for} \quad \nu = \frac{1}{2s+1} \]
Multi-Component Generalisations of Laughlin’s Wave Function

• Laughlin’s wavefunction (apart from Gaussian):

\[
\phi^L_m = \prod_{k<l}^N (z_k - z_l)^m, \quad m \text{ odd} \quad \nu = 1/m
\]

• Generalisation for SU(2) spin (Halperin, 1983):

\[
\phi^H_{m_1, m_2, n} = \prod_{k_1<l_1}^{N_1} (z^{(1)}_{k_1} - z^{(1)}_{l_1})^{m_1} \prod_{k_2<l_2}^{N_2} (z^{(2)}_{k_2} - z^{(2)}_{l_2})^{m_2} \prod_{k_1, k_2}^{N_1, N_2} (z^{(1)}_{k_1} - z^{(2)}_{k_2})^{n}
\]

• SU(\(K\)) generalisation [Wen, Zee (1991); MOG, Regnault (2007)]:

\[
\psi^{SU(K)}_{m_1, \ldots, m_K; n_{i,j}} = \prod_{j=1}^{K} \prod_{k_j<l_j}^{N_j} (z^{(j)}_{k_j} - z^{(j)}_{l_j})^{m_j} \prod_{i<j}^{K} \prod_{k_i, k_j}^{N_i, N_j} (z^{(i)}_{k_i} - z^{(j)}_{k_j})^{n_{i,j}}
\]
Graphene FQHE at $\nu_G = 1/3$ – the Problem

Recent observation of a FQHE at $\nu_G = 1/3$ (not $-2 + 1/3$ !) in suspended graphene (two-terminal measurement)

[Du et al., Nature 462, 192 (2009); Bolotin et al., ibid. 196]

\[
\begin{align*}
\text{Large } \Delta Z \Rightarrow \text{ Laughlin (Halperin) } 1/3 \text{ state in spin-\textdownarrow branch} \\
\text{BUT: } \frac{\Delta Z}{e^2/\epsilon l_B} \sim 0.002 \sqrt{B[T]} \times \epsilon \ll 1
\end{align*}
\]

Trial four-component wave function

[Papić, MOG, Regnault, PRL 105, 176802 (2010)]:

\[
\Psi_{1/3}^{SU(4)} = \prod_{i<j} \left( z_{i\downarrow} - z_{j\downarrow} \right)^3 \prod_{i<j} \left( z_{i\uparrow,K} - z_{j\uparrow,K} \right) \prod_{i<j} \left( z_{i\uparrow,K'} - z_{j\uparrow,K'} \right)
\]
Graphene FQHE at $\nu_G = 1/3$ -- Exact Diagonalisation

SU(4) spectrum ($N = 17$, $N_B = 6$)

Possible Excitations:

For $\Delta_Z > \Delta_Z^1 \simeq 0.01e^2/\epsilon l_B$: trial state stabilised

For $\Delta_Z^1 < \Delta_Z < \Delta_Z^2 \simeq 0.03e^2/\epsilon l_B$: low-energy spin-flip excitations

Papić, MOG, Regnault, PRL 105, 176802 (2010)
1/3 FQHE Family in Graphene on h-BN

(i) SU(4): $\nu_G = -2 + \nu \leftrightarrow \nu_G = 2 - \nu$

(ii) SU(2)↑ × SU(2)↓: $\nu_G = -2 + \nu \leftrightarrow \nu_G = -\nu \leftrightarrow \nu_G = \nu \leftrightarrow \nu_G = 2 - \nu$

(iii) full spin-valley symmetry breaking: $\nu_G = -2 + \nu \leftrightarrow \nu_G = -1 - \nu \leftrightarrow \nu_G = -1 + \nu \leftrightarrow \ldots$

[Dean et al., arXiv:1010.1179]
**Conclusions**

Novel physics in *graphene* as compared to the 2DEG

Weak correlations in the IQHE regime

- novel collective excitations (linear magneto-plasmons)
- visible in Bernstein modes

Strong correlations in partially filled LLs

- graphene $\simeq$ four-component quantum Hall system
  $\Rightarrow$ SU(4) theory of the FQHE
  - at $\nu_G = 1/3$ SU(4) FQHE with novel spin excitations
Correlations and Dirac points – screening

• (static) RPA screening:

\[ \epsilon^{RPA}(q) = 1 - \frac{2\pi e^2}{\epsilon q} \Pi^0(q) \]

• bare polarisability: two contributions

\[ \Pi^0(q) = \Pi^{vac}(q) + \Pi^{dop}(q) \]

• interband contribution absorbed in \( \epsilon \to \epsilon \epsilon_\infty \)

\[ \alpha_G = \frac{\alpha_G}{\epsilon_\infty} = \frac{\alpha_G}{1 + \pi \alpha_G / 2} \]
Effective SU(2) Interaction Potentials

- Broadly similar to non-relativistic case
- Largest difference between rel. and non-rel. case in $n = 1$
Haldane’s Pseudopotentials and Laughlin’s Wavefunction

• Two-particle wavefunction in $n = 0$:
  \[
  \langle z, z' | m, M \rangle \sim (z + z')^M (z - z')^m e^{-(|z|^2 + |z'|^2)/4}
  \]
  – analyticity: integral $M, m$
  – spin-polarised fermions: odd $m$

• Pseudopotentials: projection of $V$ to relative angular momentum $m$ Haldane, 1983
  \[
  V_m^{n=0} = \frac{\langle m, M | V | m, M \rangle}{\langle m, M | m, M \rangle}
  \]

• Laughlin’s wavefunction:
  \[
  \psi_s^L(\{z_j\}) = \prod_{i<j} (z_i - z_j)^{2s+1} e^{-\sum_k |z_k|^2/4}
  \]
  for $\nu = \frac{1}{2s + 1}$

⇒ Screens all pseudopotentials with $m < 2s + 1$
Pseudopotentials in Graphene

- Energies of two-particle eigenfunctions
- $m$ odd (even) $\Leftrightarrow$ spin (un)polarised
- $V(r) \sim V(c\sqrt{m})$

- Similar shape of rel. interaction in $n = 0$ and $n = 1$
  (i) relativistic $V_0^n$ smaller for $n = 1$
  (ii) $V_m^1$ smoother for relativistic case
Laughlin’s quantum-classical analogy

\[ |\psi^L_m| \sim e^{-\beta \mathcal{H}} \]

Hamiltonian of 2D one-component plasma (\( \beta \equiv 1 \))

\[ \mathcal{H} = -m \sum_{k<l} \ln |z_k - z_l| + \sum_k \frac{|z_k|^2}{4} \]

\( m \): charge of plasma particles

⇒ **General idea:**

minimisation of \( \mathcal{H} \sim \) maximal quantum probability

Example: plasma neutrality \( \Rightarrow \nu = 1/m \)
The Plasma Picture of Trial Wavefunctions (II)

• In SU($K$) case: $K$ coupled 2D plasmas
  R. de Gail, N. Regnault, MOG, PRB 77, 165310 (2008)

$$\mathcal{H}_N = - \sum_{i=1}^{K} m_i \sum_{k_i < l_i}^{N_i} \ln |z^{(i)}_{k_i} - z^{(i)}_{l_i}| - \sum_{i<j}^{K} \sum_{k_i, k_j}^{N_i} \ln |z^{(i)}_{k_i} - z^{(j)}_{l_j}| + \sum_{j=i}^{K} \sum_{k_i=1}^{N_i} \frac{|z^{(i)}_{k_j}|^2}{4}$$

• Stability criterion: $M$ must be positive (minimum in energy)

• Stability criterion for SU(2):

  $$m_1, m_2 \geq n$$

• Example 1 at $\nu = 1/2$:
  $(m_1 = 3, m_2 = 3, n = 1)$-wf homogeneous liquid
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$$\mathcal{H}_N = -\sum_{i=1}^K m_i \sum_{k_i < l_i}^{N_i} \ln |z^{(i)}_{k_i} - z^{(i)}_{l_i}| - \sum_{i<j}^K n_{ij} \sum_{k_i, k_j} \ln |z^{(i)}_{k_i} - z^{(j)}_{l_j}| + \sum_{j=i}^K \sum_{k_i=1}^{N_i} \frac{|z^{(i)}_{k_j}|^2}{4}$$

- Stability criterion: $M$ must be positive (minimum in energy)

- Stability criterion for SU(2):
  $$m_1, m_2 \geq n$$

- Example 2 at $\nu = 1/2$:
  ($m_1 = 1, m_2 = 1, n = 3$)-wf  phase separation

Scenario checked in exact-diagonalisation studies