Collective Excitations of Electrons in a Strong Magnetic Field: The Difference between Graphene and Semiconductor Heterostructures

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Nobel Prize 2010 in Physics: Graphene

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Photo: Sergeom, Wikimedia Commons

Photo: University of Manchester, UK
What is Graphene?

natural graphite

( metallic ! )

AFM image of graphene

1 μm

crystal structure of graphite

weak van der Waals bonds (between planes)

strong covalent bonds (inplane)

electric contacts
Graphene Family (sp2 Hybridisation)

sp2-hybridised carbon atom
**Outline**

- Introduction to the electronic properties of graphene
  - no magnetic field
  - with magnetic field
  - the role of electronic interactions

- Collective particle-hole excitations in the IQHE regime

- Not covered: Strong correlations and the FQHE in graphene

**Graphene bandstructure papers from the ’50ies**

- Tight-binding model ($p_z$-electron nn hopping)

\[
H_0 = -t \sum_{i \in A} \sum_{j=1}^{3} (b_R^+ e_j a_R + H.c.)
\]

reciprocal-space Hamiltonian (A-B sublattice basis):

\[
H_0 = t \begin{pmatrix}
0 & \gamma_k^* \\
\gamma_k & 0
\end{pmatrix}
\]

\[
\gamma_k = \sum_j \exp(-i k \cdot e_j)
\]
Dirac fermions

- Undoped graphene: lattice half-filled with electrons
  \[ \varepsilon_{\mathbf{K}\pm} = 0 \]:
  at \( K \) and \( K' \) points of the 1st BZ

- Continuum limit \( \mathbf{k} = \mathbf{K}\pm + \mathbf{q} \) with \( |\mathbf{q}| \ll 1/a \):

\[
\mathcal{H}^{\xi=\pm}(\mathbf{q}) = \xi \frac{3}{2} ta \begin{pmatrix} 0 & q_x - i q_y \\ q_x + i q_y & 0 \end{pmatrix} = \xi \hbar v_F \mathbf{\sigma} \cdot \mathbf{q}
\]

2D Dirac Hamiltonian for massless particles

- Energy dispersion (two-fold valley degeneracy \( \xi = \pm \)):

\[
\varepsilon_{\lambda=\pm, \xi=\pm} = \pm \hbar v_F |\mathbf{q}|
\]
Graphene in a Magnetic Field

• Magnetic field \((B\mathbf{e}_z = \nabla \times \mathbf{A})\) via Peierls substitution:

\[
\hbar \kappa \rightarrow -i\hbar \nabla + eA
\]

(semiclassical: \(\varepsilon(\kappa) \rightarrow \varepsilon(\sqrt{2n/l_B}), \quad l_B = \sqrt{\hbar/eB}\))

• Energy dispersion with magnetic field (degenerate in valley isospin \(\xi\)):

\[
\varepsilon_{\lambda,n} = \lambda \hbar \frac{v_F}{l_B} \sqrt{2|n|} \propto \sqrt{B|n|}
\]

(Relativistic LLs)

• Quantum Hall effect at

\[
\nu = \frac{n_{el}}{eB/h} = \pm 2, \pm 6, \pm 10, \ldots
\]
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IQHE in graphene

Novoselov et al., Nature 438, 197 (2005)
Zhang et al., Nature 438, 201 (2005)

Graphene IQHE:
\[ R_H = \frac{h}{e^2 \nu} \]

at \( \nu = 2(2n+1) \)

Usual IQHE:

at \( \nu = 2n \)
(no Zeeman)

\[ V_g = 15 \text{V} \]
\[ T = 30 \text{mK} \]

\[ B = 9 \text{T} \]
\[ T = 1.6 \text{K} \]

\[ R = \frac{\nu}{\nu} \]
Coulomb interaction in 2DEG

\[ r_s = \frac{e^2}{\varepsilon L \hbar^2} = \frac{1}{a_0 k_F} \propto n_{el}^{-1/2} \]

⇒ electron correlations relevant at low densities

\[ r_s^G = \frac{e^2}{\hbar \varepsilon v_F L} \approx \frac{2}{\varepsilon} \]

⇒ density-independent (fine-structure constant)
Interactions in graphene – coupling constant & screening

Coulomb interaction in 2DEG

\[ r_s = \frac{e^2}{\epsilon L \hbar^2} = \frac{1}{a_0 k_F} \propto n_{el}^{-1/2} \]

⇒ electron correlations relevant at low densities

Interaction range in 2DEG

\[ k_{TF} = r_s k_F = a_0^{-1} \]

⇒ density-independent screening length \( \lambda_{TF} \sim a_0 \)

... and in graphene

\[ r_s^G = \frac{e^2}{\hbar v_F L} = \frac{e^2}{\hbar \epsilon v_F} \sim \frac{2}{\epsilon} \]

⇒ density-independent (fine-structure constant)

... and in graphene

\[ k_{TF} \sim \frac{2k_F}{\epsilon} \sim \sqrt{n_{el}} \]

⇒ no TF screening at Dirac points, \( \lambda_{TF} \gg a \)
**Correlations in a strong magnetic field**

**Integer filling factors**

```
\begin{array}{c}
\text{relativistic LLs} \\
4 \\
3 \\
2 \\
1 \\
\end{array}
```

\[ n = 0, 1, 2, 3, 4 \]

→ perturbation expansion in

\[ r_s^G = \left( \frac{e^2}{\epsilon R_C} \right) / \Delta_n \sim \frac{e^2}{\hbar \epsilon v_F} \]

→ collective PH excitations

**Partial filling factors**

```
\begin{array}{c}
\text{relativistic LLs} \\
4 \\
3 \\
2 \\
1 \\
\end{array}
```

\[ n = 0, 1 \]

→ kinetic energy irrelevant (quenched)

→ realm of the FQHE
Collective Excitations in Doped Graphene in the IQHE regime

R. Roldán, J.-N. Fuchs, MOG,
**Particle-hole excitations in a conventional 2DEG**

Band dispersion: \( \varepsilon_k = \frac{\hbar^2 k^2}{2m_B} \)

Polarisation diagram ("electron-hole bubble"): 

\[
\Pi^0(k, \omega) = \int \frac{d\omega'}{2\pi} \sum_{k'} G(k + k', \omega + \omega')G(k', \omega')
\]
SPES of the 2DEG in a magnetic field

Landau levels (LLs): \( \varepsilon_n = \frac{\hbar e B}{m_B} \left( n + \frac{1}{2} \right) \)

LL quantisation \( \rightarrow \) weight on horizontal lines ("magneto-excitons")
Particle-hole excitations in graphene

Band dispersion: $\varepsilon_{\lambda=\pm, q} = \lambda \hbar v_F |q|$

Polarisation diagram ("electron-hole bubble"):

$$\Pi^0(q, \omega) = \sum_{\lambda, \lambda'; k} \frac{1 + \lambda \lambda' \cos(\theta_{k, q})}{2} \frac{n(\varepsilon_{\lambda', q+k}) - n(\varepsilon_{\lambda, k})}{\omega - (\varepsilon_{\lambda, k} - \varepsilon_{\lambda', q+k}) + i\delta}$$

absence of backscattering $\rightarrow$ concentration around $\omega = \hbar v_F |q|$
SPES of graphene in a magnetic field

Landau levels (LLs): \[ \varepsilon_{\lambda,n} = \frac{\hbar v_F}{l_B} \sqrt{2n} \]

- LLs no longer equidistant \([\Pi^0(\omega, q) = \text{long & boring formula}]\)
- no horizontal magneto-excitons
- weight around diagonal lines ("linear magneto-plasmons")
**Particle-hole excitations in the RPA**

Electron-hole bubble diagram in the RPA:

\[ \Pi^{RPA}(q, \omega) = \Pi^0(q, \omega)/\left[1 - v(q)\Pi^0(q, \omega)\right] \]

- appearance of 2D plasmon mode with \( \omega_{\text{plas}} \sim \sqrt{q} \)
- magneto-excitons (2DEG) acquire weak dispersion
- linear magneto-plasmons (graphene) more pronounced
Bernstein modes in high-mobility graphene

Hybridisation of plasmon mode with inter-LL transitions

Dyson-type equation (at resonance):

\[
\omega_\pm^2 = \frac{\omega_{\text{plas}}(q)^2 + \Omega_{\lambda n,n'}^2}{2} \pm \sqrt{\frac{[\omega_{\text{plas}}^2 - \Omega_{\lambda n,n'}^2]^2}{4} + 4 \left( \frac{e^2}{eql_B^2} \right)^2 \omega_{\text{plas}} \Omega_{\lambda n,n'} |F_{\lambda n,n'}(q)|^2}
\]
Conclusions

Novel physics in *graphene* as compared to the 2DEG

Weak correlations in the IQHE regime

- relativistic carriers $\rightarrow$ particular particle-hole excitation spectrum
- absence of backscattering
- novel collective excitations (linear magneto-plasmons)
Interplay between 2D Dirac physics and strong correlations

- how to make graphene “more correlated”? (problem of the vanishing DOS)
- organic materials: $\alpha$-(BEDT-TTF)$_2$I$_3$ (larger value of $r_s$)
- cold atoms in optical honeycomb lattice with true short-range interactions
Correlations and Dirac points – screening

- (static) RPA screening:

\[ \epsilon^{\text{RPA}}(q) = 1 - \frac{2\pi e^2}{\epsilon q} \Pi^0(q) \]

- bare polarisability: two contributions

\[ \Pi^0(q) = \Pi^\text{vac}(q) + \Pi^\text{dop}(q) \]

- interband contribution absorbed in \( \epsilon \to \epsilon \epsilon_{\infty} \)

\[ r_s^* = \frac{r_s}{\epsilon_{\infty}} = \frac{r_s}{1 + \pi r_s / 2} \]