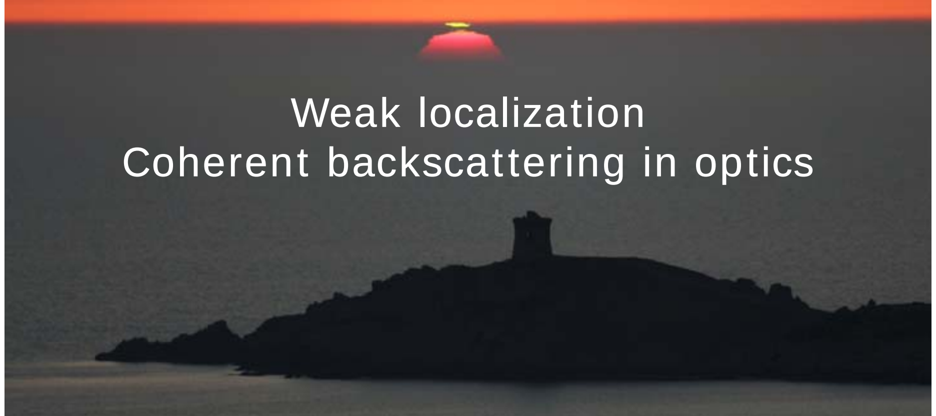
Disorder and mesoscopic physics

Lecture 3

Weak localization
Coherent backscattering in optics



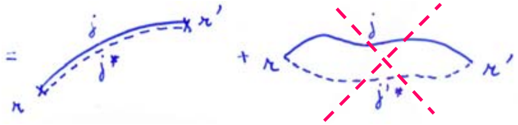
Gilles Montambaux, Université Paris-Sud, Orsay, France

users.lps.u-psud.fr/montambaux

Summary of previous lecture

conductance ~ transmission ~ probability

$P(r,r')$ =



quantum crossing → 1 / g correction

$$p_\times(\tau_D) \sim \frac{\lambda_F^{d-1} v_F \tau_D}{V} \sim \frac{1}{g}$$

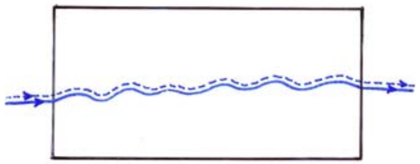
classical transport $\propto g \frac{e^2}{h}$

quantum effects $\propto \frac{e^2}{h}$

Weak localization

Classical conductance

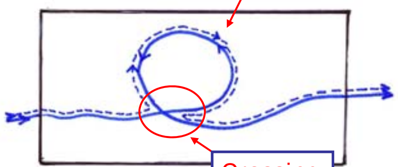
G_{cl}



$\propto P_{cl}(0,L)$

Quantum correction ⇒ **One crossing** ⇒ **One loop**

$$\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle$$



$\propto \Delta P(0,L)$

$P_{\text{int}}(t)$ = distribution of number of loops with time t = return probabililty

$$\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle$$

Weak-Localization

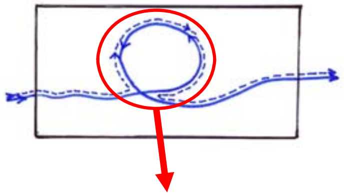
Nb loops and return probability
Magnetic field, phase coherence
Weak-localisation in dimension d

A few solutions of the diffusion equation and WL
- Magnetic field and negative magnetoresistance
- Magnetic field in quasi-1D wires
- AAS oscillations

Weak localization : how to calculate $P_{\text{int}}(t)$?

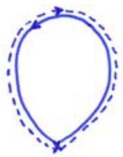
$$\Delta G \sim -\frac{2e^2}{h} \langle P_{\text{int}}(t) \rangle$$

$$P_{\text{int}}(t) = \int P_{\text{int}}(r,r,t) d^d r$$



$P_{\text{int}}(t)$

Cooperon



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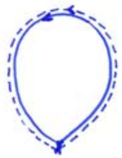


$P_{cl}(t)$

Diffuson

Interference term

Classical return probability

$$P_{\text{int}}(r,r,t) = P_{cl}(r,r,t)$$

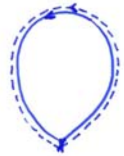
If time reversal invariance

Important difference :

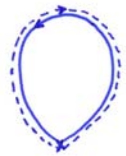
$P_{cl}(r,r',t)$ $\Rightarrow$ paired trajectories follow the same direction

$P_{\text{int}}(r,r',t)$ $\Rightarrow$ paired trajectories follow opposite directions

Diffuson

A_j  A_j^*

Cooperon

A_j  A_j^{*T}

A_j A_j^T have the same phase $\int \vec{p}.\vec{dl}$

$$P_{\text{int}}(r,r,t) = P_{cl}(r,r,t)$$

If time reversal invariance

If phase coherence between the reversed trajectories is preserved

Weak localization

$$\Delta G \sim -\frac{2e^2}{h} \langle P_{\text{int}}(t) \rangle = -\frac{2e^2}{h} \int_{\tau_e}^{\tau_\phi} P_{\text{int}}(t) \frac{dt}{\tau_D}$$

$\tau_\phi = \frac{L_\phi^2}{D}$ phase coherence time

τ_e elastic collision time

$\tau_D = \frac{L^2}{D}$ time spent in the sample

for $t < \tau_D$

$$P_{\text{int}}(t) = P_{cl}(t) = \frac{L^d}{(4\pi D t)^{d/2}}$$

volume explored after time t

The return probability $P(t)$ increases for small d

Coherent effets are more important in low dimension

Weak localization correction : exact result

Qualitative result

$$\Delta G \sim -2\frac{e^2}{h} \langle P_{\text{int}}(t) \rangle \sim -2\frac{e^2}{h} \int_{\tau_e}^{\tau_\phi} P_{\text{int}}(t) \frac{dt}{\tau_D}$$

correct result

$$\Delta G = -4\frac{e^2}{h} \int_0^\infty P_{\text{int}}(t) \left(e^{-t/\tau_\phi} - e^{-t/\tau_e} \right) \frac{dt}{\tau_D}$$

Long trajectories are cut because of loss of phase coherence beyond τ_ϕ

Measurement of this quantum correction gives access to the coherence length

Weak localization : dependence on dimensionality

$$\Delta G = -4\frac{e^2}{h}\int_0^{\infty} P_{\text{int}}(t)\left(e^{-t/\tau_\phi} - e^{-t/\tau_e}\right)\frac{dt}{\tau_D}$$

Macroscopic limit $L \gg L_\phi \quad \tau_D \gg \tau_\phi$

$$P(t) = \frac{V}{(4\pi D t)^{d/2}}$$

$$\int_{\tau_e}^{\tau_\phi} \frac{dt}{t^{d/2}} \rightarrow \begin{cases} \sqrt{\tau_\phi} - \sqrt{\tau_e} & d=1 \quad (quasi-1D) \\ \ln\frac{\tau_\phi}{\tau_e} & d=2 \\ \frac{1}{\sqrt{\tau_e}} - \frac{1}{\sqrt{\tau_\phi}} & d=3 \end{cases}$$

Weak localization : dependence on dimensionality

$$\Delta G = -s\frac{e^2}{h}\frac{L_\phi(T)}{L} \qquad d=1 \quad (quasi-1D)$$

$$\Delta G = -s\frac{e^2}{\pi h}\ln\frac{L_\phi(T)}{l_e} \qquad d=2$$

$$\Delta G = -s\frac{e^2}{2\pi h}\frac{L}{l_e} \qquad d=3$$

Correction more important for small d because return probability is enhanced

Mesoscopic system $L \ll L_\phi \qquad \tau_D \ll \tau_\phi$

$$\Delta G \sim -s\frac{e^2}{h} \qquad d=1 \quad (quasi-1D)$$

$$\Delta G \sim -s\frac{e^2}{\pi h}\ln\frac{L}{l_e} \qquad d=2$$

$$\Delta G \sim -s\frac{e^2}{2\pi h}\frac{L}{l_e} \qquad d=3$$

Weak localization : dependence on dimensionality

$$d=1 \quad \Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L}$$

$$d=2 \quad \Delta g = -\frac{1}{\pi}\ln\frac{L_\phi(T)}{l_e} \qquad g = \frac{k_F l_e}{2}$$

$$d=3 \quad \Delta g = -\frac{1}{2\pi}\frac{L}{l_e} \qquad g = \frac{k_F^2 l_e L}{3\pi}$$

Correction more important for small d because return probability is enhanced

$$g = \frac{A_d}{(2\pi)^{d-1}}(k_F W)^{d-1}\frac{l_e}{L}$$

Weak localization : dependence on dimensionality

$$d=1 \quad \Delta g = -\frac{L_\phi(T)}{L} \qquad g = M\frac{l_e}{L} \qquad \frac{\Delta g}{g} \sim \frac{L_\phi}{M\, l_e}$$

$$d=2 \quad \Delta g = -\frac{1}{\pi}\ln\frac{L_\phi(T)}{l_e} \qquad g = \frac{k_F l_e}{2} \qquad \frac{\Delta g}{g} = \frac{2}{\pi}\frac{\ln(L_\phi / l_e)}{k_F l_e}$$

$$d=3 \quad \Delta g = -\frac{1}{2\pi}\frac{L}{l_e} \qquad g = \frac{k_F^2 l_e L}{3\pi} \qquad \frac{\Delta g}{g} \propto \frac{1}{k_F^2 l_e^{\,2}}$$

$\frac{\Delta g}{g} \sim 1$ defines a new length scale at which perturbation breaks down

Localization length : $\xi_{1D} \sim M\, l_e$

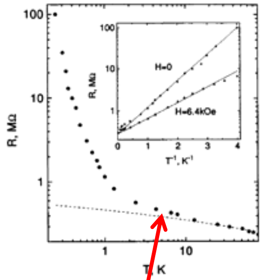
$$\xi_{2D} \sim l_e e^{\frac{\pi}{2}k_F l_e}$$

$$g = \frac{A_d}{(2\pi)^{d-1}}(k_F W)^{d-1}\frac{l_e}{L}$$

$d = 1 \quad (quasi-1D)$

$$\Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L} \qquad \frac{\Delta g}{g} = -\frac{L_\phi}{M\, l_e}$$

Localization length $\xi = M\, l_e$



$M \sim 10 \qquad l_e \sim 20nm \qquad L_\phi \sim \xi$

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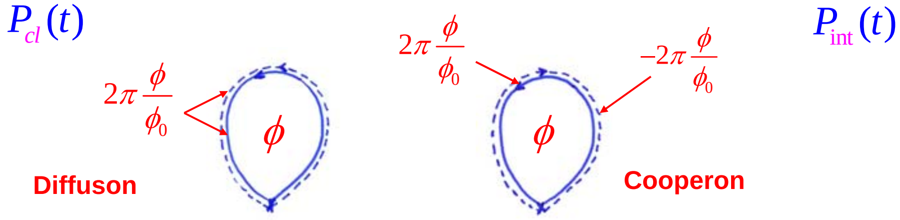
M.E. Gershenson et al.

Crossover from Weak to Strong Localization in Quasi-One-Dimensional Conductors

The crossover from weak to strong localization in the resistance of quasi-1D conductors is observed for the first time with decreasing the temperature; it occurs when the phase-breaking length becomes comparable with the localization length. The signature of the strong-localization regime is an activation-type temperature dependence of the resistance and exponentially strong negative magnetoresistance. The magnetoresistance is well described by the theory of doubling of the localization length in quasi-1D conductors in strong fields; this provides a direct measurement of the localization length.

Phase coherence and magnetic field

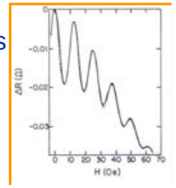


Cooperon: in a magnetic flux, paired trajectories get opposite phases

→ phase difference $4\pi\frac{\phi}{\phi_0}$ → Oscillations of period $\frac{\phi_0}{2} = \frac{h}{2e}$

In a magnetic field, dephasing between time reversed trajectories
→The cooperon oscillates with flux

→ It cancels in a magnetic field

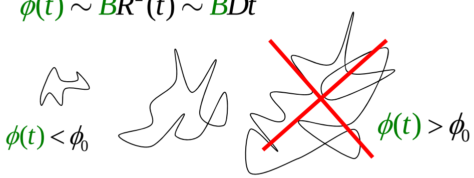


Sharvin, Sharvin

Effect of magnetic field (qualitative)

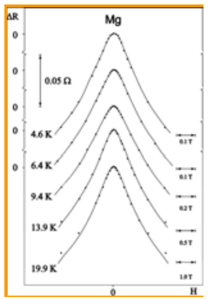
$$P_{int}(t) = P_{cl}(t)\left\langle e^{4i\pi\frac{\phi(t)}{\phi_0}}\right\rangle \sim e^{-t/\tau_B}$$

$\phi(t) \sim BR^2(t) \sim BDt$

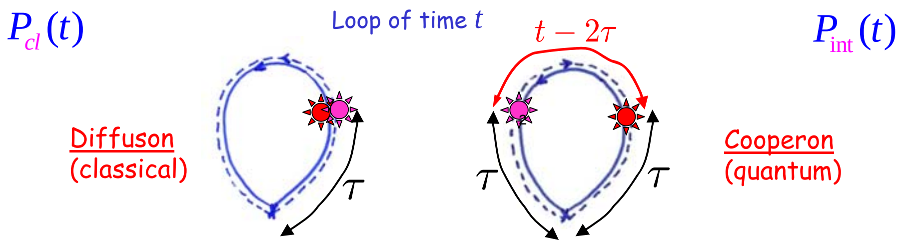


$$BD\tau_B = \phi_0$$

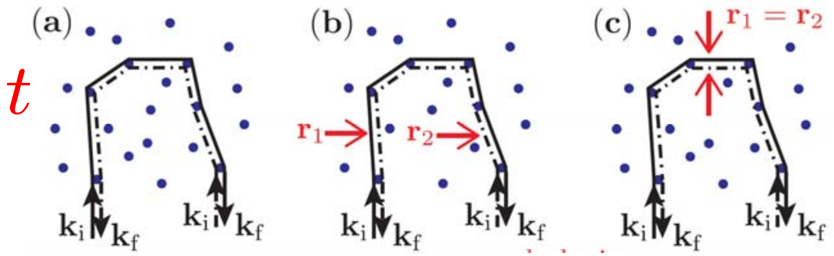
Trajectories which enclose more than one flux quantum do not contribute to $P_{int}(t)$



Weak-localization = phase coherence



Weak-localization = phase coherence



$\tau \neq \frac{t}{2}$ « Suppression »

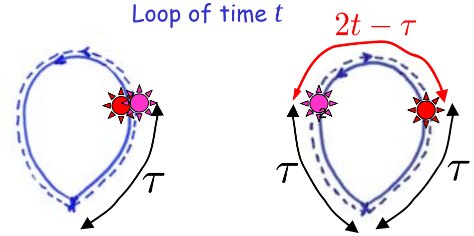
$\tau = \frac{t}{2}$ « Revival »

Suppression and revival of WL through control of time-reversal symmetry
Vincent Josse et al. , Institut d'Optique, PRL 2015

Weak-localization = phase coherence

$P_{cl}(t)$ $P_{int}(t)$

Diffuson (classical)



Cooperon (quantum)

Phase coherence broken after a typical time τ_ϕ
Only trajectories of time $t < \tau_\phi$ contribute to the return probablity and to the WL

$$P_{int}(t) = P_{cl}(t) e^{-t/\tau_\phi} e^{4i\pi\frac{\phi}{\phi_0}}$$

Magnetic impurities, e-e interaction, magnetic impurities
Altshuler, Aronov, Khmelnitskii

Diffusion equation for $P_{int}(r,r,t)$?

$$\left[\frac{1}{\tau_\phi} + \frac{\partial}{\partial t} - D\left(\nabla + i2e\frac{\vec{A}}{\hbar}\right)^2\right] P_{int}(r,r',t) = \delta(r-r')\delta(t)$$

Phase coherence time

Effective charge

Vector potential

$$r = r'$$

$$\left(\frac{\partial}{\partial t} - D\Delta\right) P_{cl}(r,r',t) = \delta(r-r')\delta(t)$$

Solving diffusion equation

$$P(t) = \sum_n e^{-E_n t}$$

where E_n are the eigenvalues of the diffusion equation $-D\left(\nabla + i2e\frac{\vec{A}}{\hbar}\right)^2 \psi_n = E_n \psi_n$

Example : uniform magnetic field in 2D

$$E_n = \left(n + \frac{1}{2}\right)\frac{4eBD}{\hbar}$$

$$P(t) = \frac{BS/\phi_0}{\sinh 4\pi BDt/\phi_0}$$

$B \to 0$: $P(t) = \frac{S}{4\pi Dt}$

$B \to \infty$: $P(t) \propto e^{-\frac{4\pi BD}{\phi_0}t}$

Four examples

$$\frac{\Delta G}{G_{cl}} \propto -\int_{\tau_e}^{\infty} P(t)\, e^{-t/\tau_\phi}\, dt$$

weak localization in 2 D, negative magnetoresistance

weak localization in a quasi-1d wire

weak localization in a ring

weak localization in a cylinder

Example 1: weak localization in 2 D

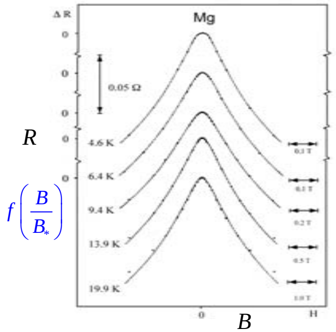
$$\frac{\Delta G}{G_{cl}} \propto -\int_{\tau_e}^{\infty} P(t)\, e^{-t/\tau_\phi}\, dt$$

$$P(t) = \frac{S}{4\pi D t} \quad \Longrightarrow \quad \Delta G \sim -\ln \frac{L_\phi}{l_e}$$

In a magnetic field :

$$P(t) = \frac{BS/\phi_0}{\sinh 4\pi B D t/\phi_0} \quad \Longrightarrow \quad \Delta G \sim -\ln \frac{\min\left(L_\phi, L_B\right)}{l_e} \qquad BL_B^2 \sim \phi_0$$

Bergmann, 84



Weak localization correction is suppressed when

$$L_B \sim L_\phi$$

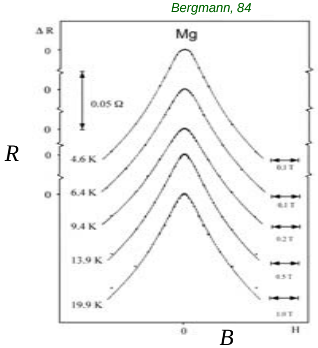
$$B_* L_\phi^2 \sim \phi_0 \quad \Longrightarrow \quad L_\phi(T) \propto 1/\sqrt{T}$$

Example 1: weak localization in 2 D

$$\Delta G = -4\frac{e^2}{h}\int_0^{\infty} \frac{B/\phi_0}{\sinh 4\pi B D t/\phi_0}\left(e^{-t/\tau_\phi} - e^{-t/\tau_e}\right)\frac{dt}{\tau_D}$$

$$\Delta G = -2\frac{e^2}{2\pi h}\left[\psi\left(\frac{1}{2}+\frac{\hbar}{4eBD\tau_e}\right) - \psi\left(\frac{1}{2}+\frac{\hbar}{4eBD\tau_\phi}\right)\right]$$

Bergmann, 84



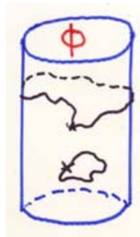
Example 4 : weak localization in a cylinder

Altshuler, Aronov, Spivak, '81

$$\frac{\Delta G}{G_{cl}} \propto -\int_{\tau_e}^{\infty} P(t)\, e^{-t/\tau_\phi}\, dt$$

Cylinder in a Aharonov-Bohm flux :

$$P(t) = \sum_m \frac{e^{-m^2 L^2/4Dt}}{4\pi D t}\cos 4\pi m \frac{\phi}{\phi_0}\, e^{-t/\tau_\phi}$$

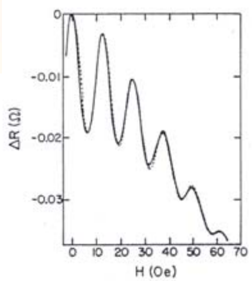


$$\sim e^{-m\frac{L}{L_\phi}}$$

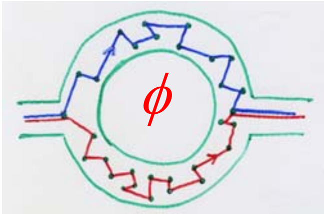
$$\Delta G_m = -s\frac{e^2}{\pi h}\left[\ln\frac{L_\phi}{l_e} + 2\sum_m K_0\left(m\frac{L}{L_\phi}\right)\cos 4\pi m \frac{\phi}{\phi_0}\right]$$

2D diffusion **winding of trajectories**

Altshuler, Aronov, Spivak, '81



Sharvin,Sharvin, '81

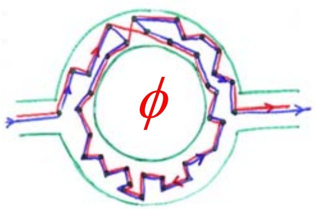


"Sample specific" interference

Phase difference between two trajectories $2\pi \frac{\phi}{\phi_0}$

→ Oscillations of period $\phi_0 = h/e$

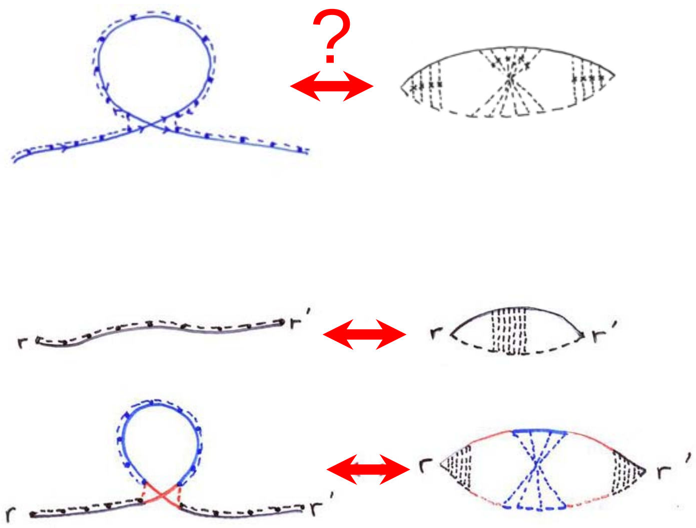
… which disappear in average

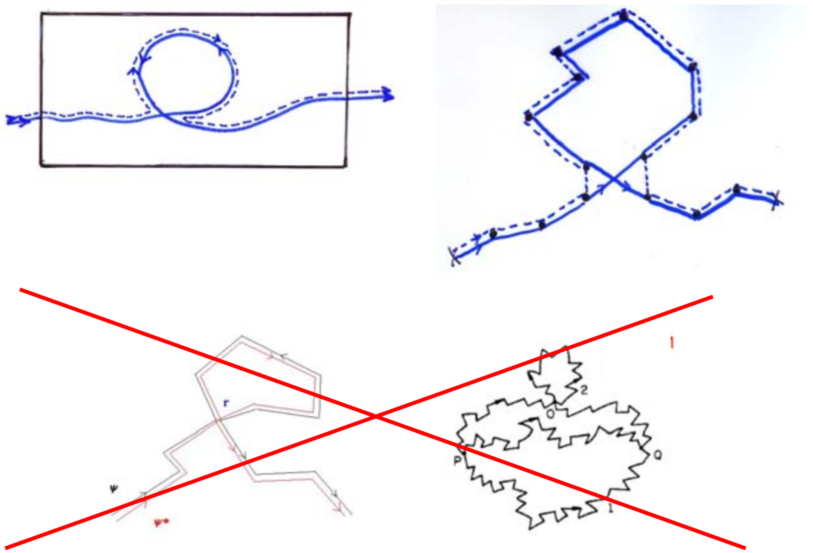


After disorder averaging, only remains
The contribution of paired trajectories

Phase difference $4\pi \frac{\phi}{\phi_0}$

→ Oscillations of period $\phi_0 / 2 = h/2e$





Summary

$$\Delta g = -2\int_0^\infty P_{\text{int}}^B(t)\left(e^{-t/\tau_\phi} - e^{-t/\tau_e}\right)\frac{dt}{\tau_D}$$

Contributions of closed diffusion trajectories whose size is limited by
Size of the system, phase coherence, magnetic field, etc.

$$\Delta g \sim -2 \int_0^{\min(\tau_D,\tau_\phi,\tau_B)} \left(\frac{\tau_D}{4\pi t}\right)^{d/2} \frac{dt}{\tau_D}$$

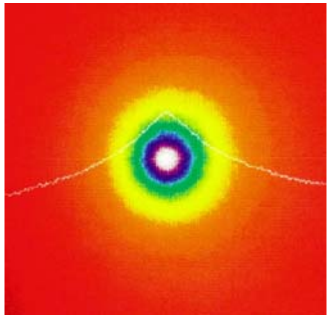
$$\tau_c \sim \min(\tau_D, \tau_\phi, \tau_B)$$

$$\Delta g = -\frac{L_c(T)}{L} \qquad d = 1 \quad (quasi-1D)$$

$$\Delta g = -\frac{1}{\pi}\ln\frac{L_c(T)}{l_e} \qquad d = 2$$

$$L_c = \sqrt{D\tau_c}$$

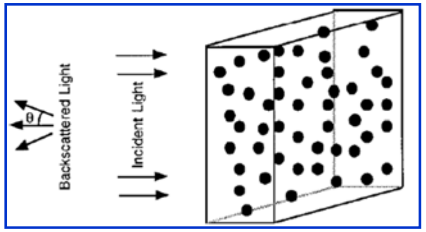
Coherent backscattering

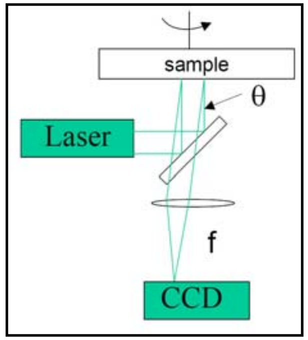


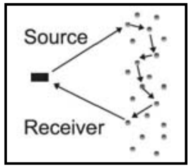
G. Maret, constance

Multiple scattering in optics

Albedo : reflexion coefficient of a scattering medium

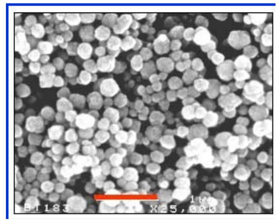






Multiple scattering of light by impurities

Example : TiO_2 powder

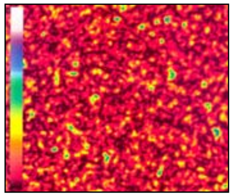


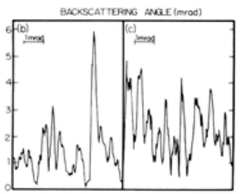
Turbid media: colloids, milk, powders, chalk, teflon, etc…

Angular dependence of the reflected intensity

speckle

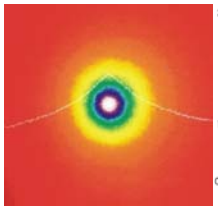
one disorder configuration

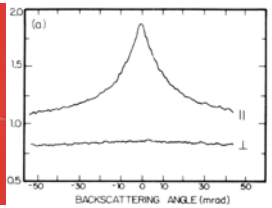




angular dependence

Average





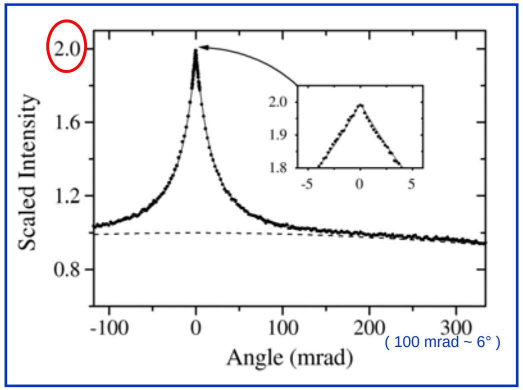
G. Maret, Constance

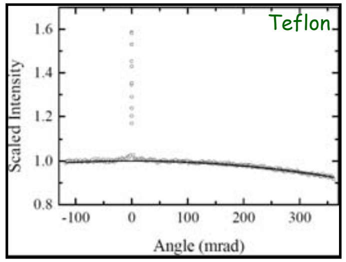
In average,
the intensity is uniform,
except near the
backscattering direction

→ There is a mechanism which survives disorder average

« Coherent backscattering »

Angular dependence of the reflected intensity





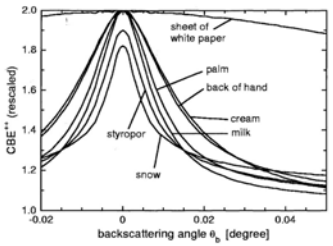
Coherent backscattering « cone »

The intensity is doubled in the backscattering direction

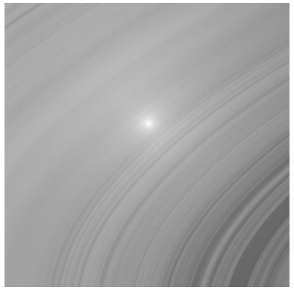
Triangular singularity

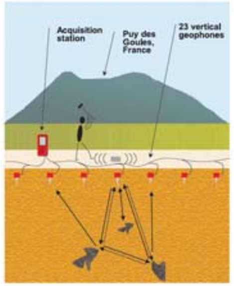
ZnO powder, D. Wiersma 1995

An ubiquitous phenomenon

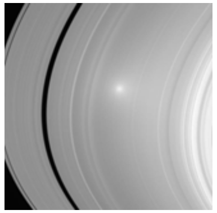


Also teflon, TiO_2, chalk, etc...



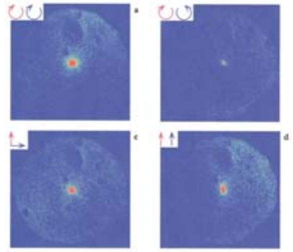


multiple scattering of microwaves, acoustic waves, sismic waves, etc.



Saturn rings,
Cassini 2006, NASA

CBS of light by cold atoms



VOLUME 83, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1999

Coherent Backscattering of Light by Cold Atoms

G. Labeyrie, F. de Tomasi,* J.-C. Bernard, C. A. Müller, C. Miniatura, and R. Kaiser

The measured CBS cone probes the internal degrees of freedom of the atoms

PHYSICAL REVIEW A, VOLUME 64, 053804

Weak localization of light by cold atoms: The impact of quantum internal structure

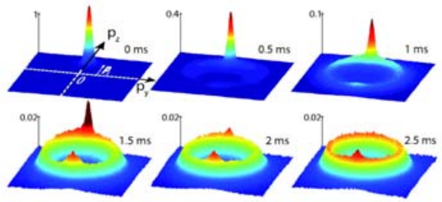
Cord A. Müller,[1,3] Thibaut Jonckheere,[2] Christian Miniatura,[1] and Dominique Delande[2,3]

CBS of cold atoms by light

Selected for a Viewpoint in Physics
PRL 109, 195302 (2012) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2012

Coherent Backscattering of Ultracold Atoms

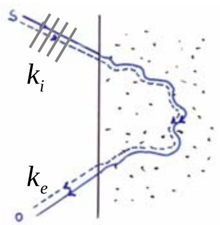
F. Jendrzejewski,[1] K. Müller,[1] J. Richard,[1] A. Date,[1] T. Plisson,[1] P. Bouyer,[2] A. Aspect,[1] and V. Josse[1,*]



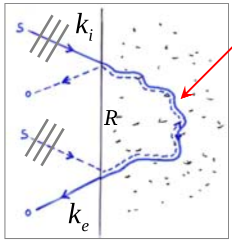
Time evolution of the momentum distribution

c.f. seminar Vincent Josse

Angular dependence of the reflected intensity



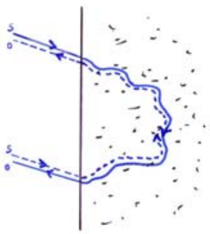
Classical diffusion



Coherent contribution

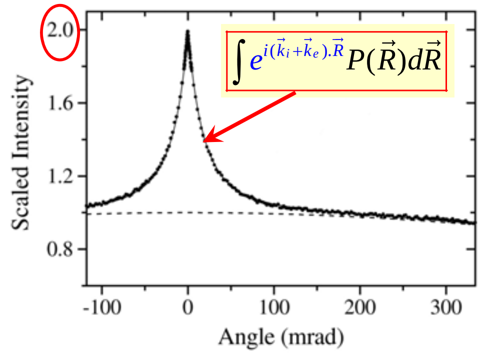
Reversed trajectories

dephasing $e^{i(\vec{k}_i+\vec{k}_e).\vec{R}}$



$$I = I_0\left[1+\left\langle\cos(\vec{k}_i+\vec{k}_e).\vec{R}\right\rangle_{\vec{R}}\right]$$

The coherent contribution is maximal if k_e=-k_i



$$\int e^{i(\vec{k}_i+\vec{k}_e).\vec{R}}P(\vec{R})d\vec{R}$$

$P(\vec{R})$ distribution of distances $\vec{R}$ between the first and the last scatterer

$$r(\theta) = \frac{1}{(1+k\,l_e\,|\theta|)^2}$$

$$\Delta\theta \propto \frac{\lambda}{l_e}$$

Measure of the mean free path from the width of the cone

Coherent backscattering « Cone »

Douling of the intensity in the backscattering direction

Triangular singularity

Conf Proc IEEE Eng Med Biol Soc. 2004;7:5285-8.

Coherent backscattering spectroscopy: a new technique for tissue diagnosis.

Kim Y[1], Liu Y, Backman V.

Abstract

Coherent backscattering (CBS) is a photon weak-localization phenomenon that gives rise to an enhanced backscattering of light by random media. This effect has been previously investigated using coherent light sources. Here we show that CBS can be observed using broadband low-coherence light. We demonstrate, for the first time to the best of our knowledge, that low-coherence detection substantially simplifies CBS measurements in biological tissue and enables depth-resolved spectroscopic analysis of CBS. CBS spectroscopy may find important applications in probing random media such as biological tissues where depth-selective measurements are crucial.

Sci Rep. 2014 Dec 1;4:7257. doi: 10.1038/srep07257.

Ultrasensitive and fast detection of denaturation of milk by coherent backscattering of light.

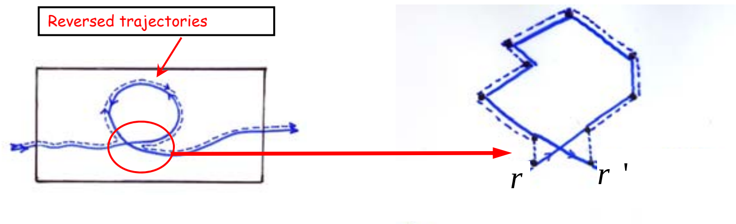
Verma M[1], Singh DK[2], Senthilkumaran P[3], Joseph J[3], Kandpal HC[2].

Abstract

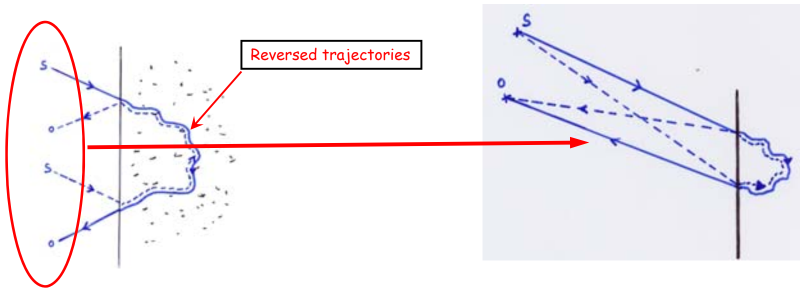
In this work, Coherence backscattering (CBS) of light has been used to detect the onset of denaturation of milk. The CBS cone shape and its enhancement factor are found to be highly sensitive to the physical state of the milk particles. The onset of denaturing of milk not visible to the naked eye, can be easily detected from changes in the CBS cone shape. The onset of denaturation is confirmed by spectral changes in Raman spectra from these milk samples. Further, the possibility to estimate the dilution of milk by water as an adulterant is demonstrated. The method reported has a broad scope in industry for making an inline ultrafast cost effective sensor for milk quality monitoring during production and before consumption.

Where is the quantum crossing ?

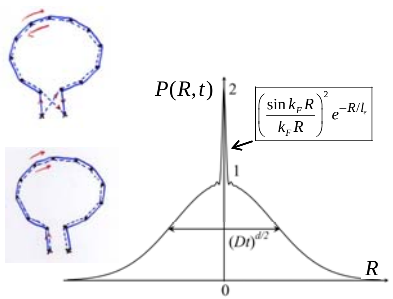
WL

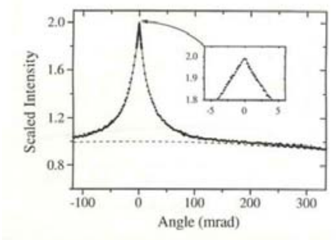


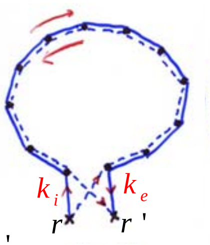
CBS



Where is the quantum crossing ?

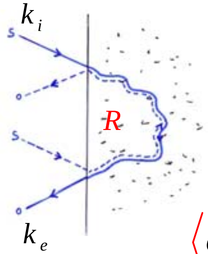






$R = r - r'$

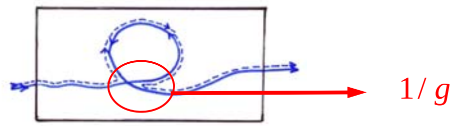
$$\left\langle e^{i(\vec{k}_i+\vec{k}_e).\vec{R}} \right\rangle_{\vec{k}_i,\vec{k}_e}$$



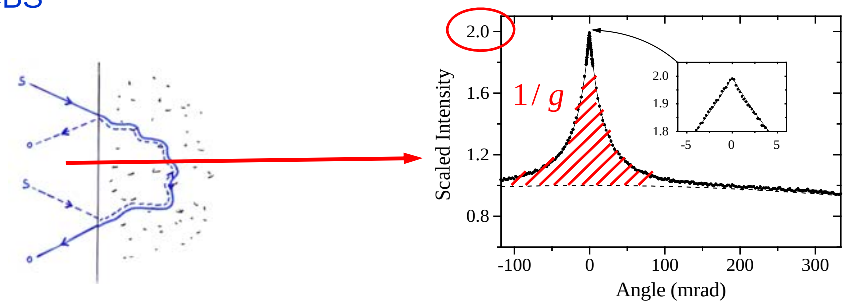
$$\left\langle e^{i(\vec{k}_i+\vec{k}_e).\vec{R}} \right\rangle_{\vec{R}}$$

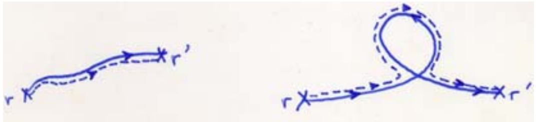
Where is the quantum crossing ?

WL



CBS



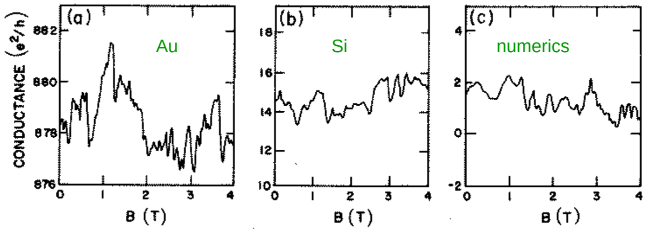


$$\bar{G} = G_{cl} + \Delta G$$

Correlations ? $\overline{GG}$

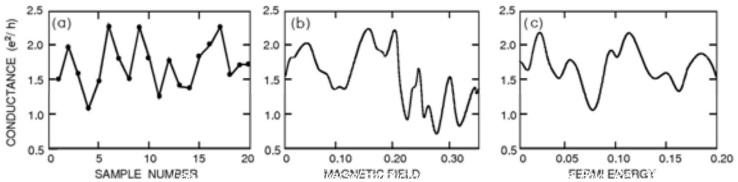
Universal conductance fluctuations

Conductance versus an external parameter (magnetic field)



Reproducible fluctuations

Universal conductance fluctuations



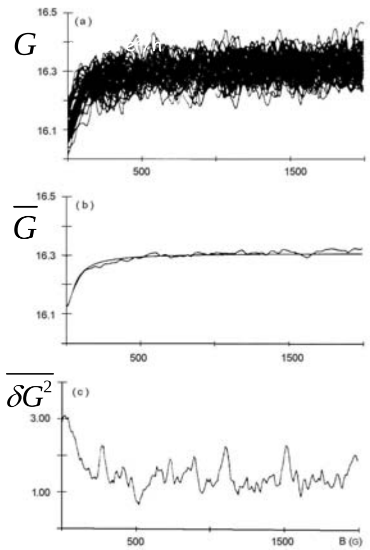
The amplitude is universal $\delta G = \sqrt{\overline{\delta G^2}} = \sqrt{\overline{G^2} - \overline{G}^2} \sim \frac{e^2}{h}$

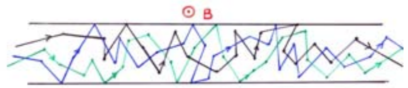
$$\frac{h}{e^2} = 25800\,\Omega$$

If quantum coherence : $L \ll L_\phi$

In a good metal, $\delta G << G$

Magneto-fingerprints





each trace represents an interference pattern

Average $\bar{G} = G_{cl} + \Delta G$

The WL correction ΔG is suppressed by the magnetic field

Variance $\overline{\delta G^2}$

In a field, the variance is reduced by a factor 2

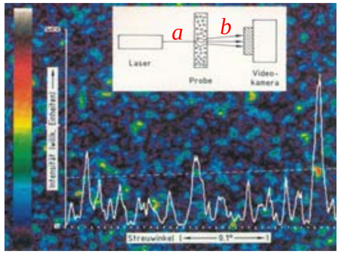
D. Mailly, M. Sanquer, J. Physique I, 2, 357 (1992)

Quantum transport in disordered systems

Conductance - Transmission

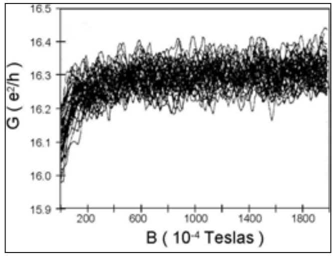
Analogies with optics

Speckle pattern : optical intensity emerging from a disordered medium



$$\delta T_{ab} = \overline{T_{ab}}$$

Conductance fluctuations



$$\delta G \ll \overline{G}$$

Quantum corrections to classical transport ~ interference effects in optics