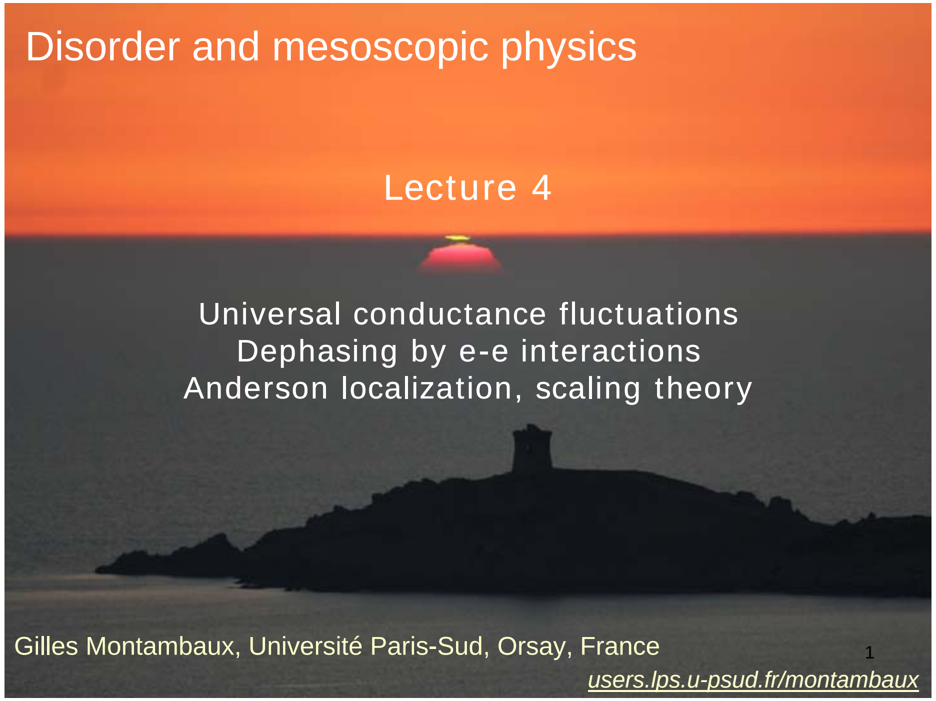


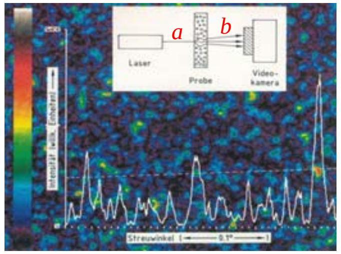
Disorder and mesoscopic physics

Lecture 4

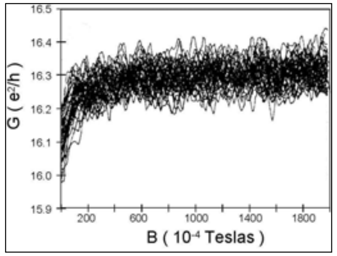
Universal conductance fluctuations
Dephasing by e-e interactions
Anderson localization, scaling theory

Gilles Montambaux, Université Paris-Sud, Orsay, France

users.lps.u-psud.fr/montambaux



$\delta T_{ab} = \overline{T_{ab}}$



$\sim \dfrac{e^2}{h}$

$\delta G \ll \overline{G}$

Origin of the universality

Analogies and differences with speckle in optics

Landauer formula

Universal conductance fluctuations : classical description

Contribution of N incoherent elements : $N = \left(\dfrac{L}{L_c}\right)^d$

$\dfrac{\delta G}{G} \sim \dfrac{1}{\sqrt{N}} \propto \dfrac{1}{L^{d/2}}$

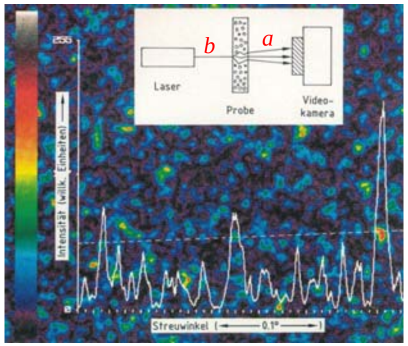
Ohm's law: $G = \sigma L^{d-2}$

$\delta G \sim \dfrac{1}{L^{(4-d)/2}} \longrightarrow$ 0 !!

→ Quantum coherence $\delta G \sim \dfrac{e^2}{h}$ $L \ll L_\phi$

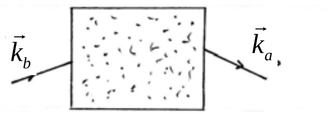
Speckle fluctuations

G. Maret



T_{ab}

Transmission from on direction ***b*** to a direction ***a***

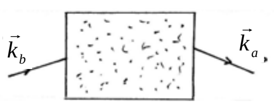


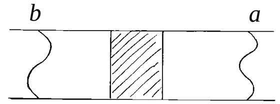
Relative fluctuations are large, of order 1

$\overline{\delta T_{ab}^2} = \overline{T_{ab}}^2$ **Rayleigh's law**

Compare with universal conductance fluctuations ? $\delta G \ll G$

Conductance = transmission





Multichannel Landauer formula

$$G = \frac{2e^2}{h} T = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

b, a : incoming and outgoing channels

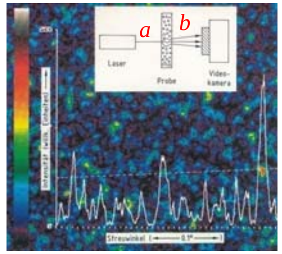
Number of channels : $M = \dfrac{k_F^2 S}{4\pi}$

Conductance = transmission

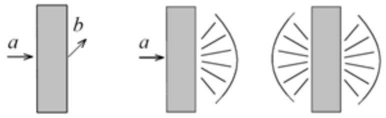
Landauer

$$G = \frac{2e^2}{h} T = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

b, a : incoming and outgoing channels



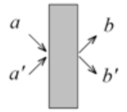
Optics : measure T_{ab} , T_a , or T



T_{ab} $\quad T_a = \sum_b T_{ab}$ $\quad T = \sum_{a,b} T_{ab}$

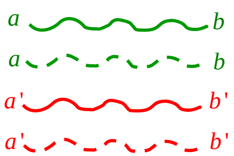
Electronics : measure $T = \sum_{a,b} T_{ab}$

$$\overline{T}_{ab} = \frac{\overline{g}}{M^2}$$



$\overline{T_{ab} T_{a'b'}}$

Conductance and speckle fluctuations



$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

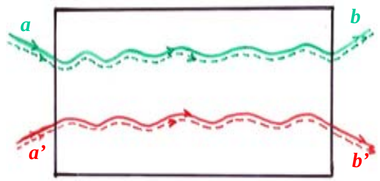
$$\overline{T_{ab} T_{a'b'}} = \overline{T_{ab}}\ \overline{T_{a'b'}} + \overline{\delta T_{ab} \delta T_{a'b'}}$$

$$\overline{\delta g^2} = \overline{g^2} - \overline{g}^2 = \sum_{a,a',b,b'} \overline{\delta T_{ab} \delta T_{a'b'}}$$

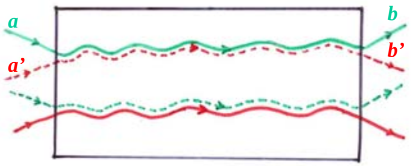
Conductance and speckle fluctuations $\quad C_1$

$$\overline{T_{ab} T_{a'b'}} = \overline{T_{ab}}\ \overline{T_{a'b'}} + \overline{\delta T_{ab} \delta T_{a'b'}}$$

$\overline{T_{ab}}\ \overline{T_{a'b'}}$

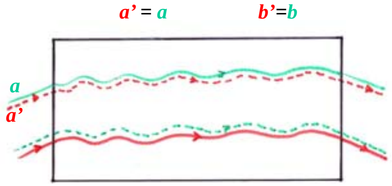


$\overline{\delta T_{ab} \delta T_{a'b'}}$



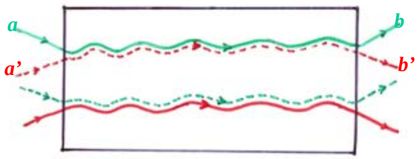
$$= \overline{T_{ab}}\ \overline{T_{a'b'}}\ \delta_{a,a'}\ \delta_{b,b'}$$

Conductance and speckle fluctuations C_1



$$\overline{\delta T_{ab}\delta T_{ab}} = \overline{T_{ab}}^{\,2}$$

Rayleigh's law



$$= \overline{T_{ab}}\;\overline{T_{a'b'}}\;\delta_{a,a'}\,\delta_{b,b'}$$

$$\overline{\delta g^2} = \sum_{a,a',b,b'} \overline{\delta T_{ab}\delta T_{a'b'}}$$

$$\overline{\delta g^2} = \sum_{a,a',b,b'} \overline{T_{ab}}\;\overline{T_{a'b'}}\;\delta_{a,a'}\,\delta_{b,b'}$$

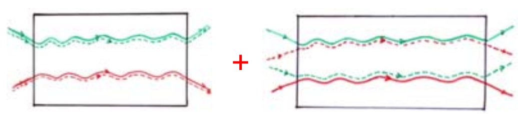
$$= \frac{g^2}{M^4} M^2 = \frac{g^2}{M^2} \sim \left(\frac{l_e}{L}\right)^2 \ll 1$$

Does not explain conductance fluctuations …

Conductance and speckle fluctuations C_1

$$\overline{T_{ab}T_{a'b'}}$$

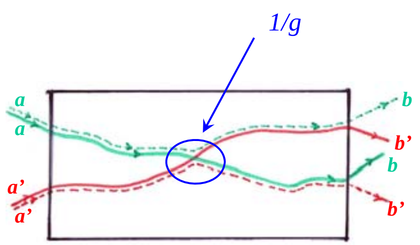
$$\overline{T_{ab}}\;\overline{T_{a'b'}} \quad + \quad \overline{\delta T_{ab}\delta T_{a'b'}}$$



+ ???

$$\overline{T_{ab}}\;\overline{T_{a'b'}}\;\delta_{a,a'}\,\delta_{b,b'}$$

Conductance and speckle fluctuations C_2



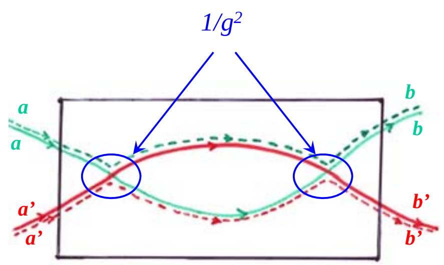
$$\overline{\delta T_{ab}\delta T_{a'b'}} = \frac{2}{3g}\overline{T_{ab}}\;\overline{T_{a'b'}}\;\delta_{b,b'}\;\left[+\delta_{a,a'}\right]$$

$$\overline{\delta g^2} = \frac{2}{3g}\sum_{a,a',b,b'} \overline{T_{ab}}\;\overline{T_{a'b'}}\;\left[\delta_{a,a'} + \delta_{b,b'}\right]$$

$$= \frac{4}{3g}\frac{g^2}{M^4}M^3 = \frac{4}{3}\frac{g}{M} \sim \frac{l_e}{L} \ll 1$$

Does not explain conductance fluctuations…

Conductance and speckle fluctuations C_3

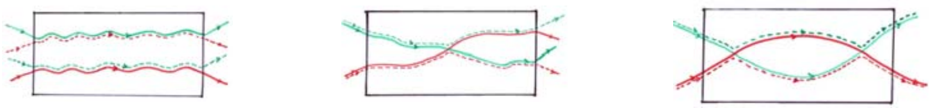


$$\overline{\delta T_{ab}\delta T_{a'b'}} = \frac{2}{15g^2}\overline{T_{ab}}\;\overline{T_{a'b'}}$$

$$\overline{\delta g^2} = \frac{2}{15g^2}\sum_{a,a',b,b'} \overline{T_{ab}}\;\overline{T_{a'b'}} \qquad = \frac{2}{15g^2}\frac{g^2}{M^4}M^4 = \frac{2}{15}$$

Universal conductance fluctuations !

Conductance and speckle fluctuations — summary



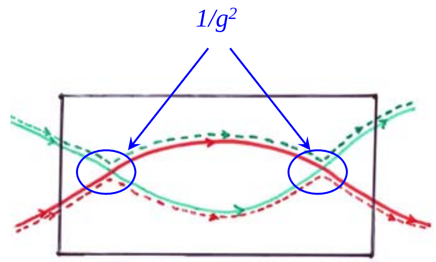
C$_1$ C$_2$ C$_3$

$$\overline{\delta T_{ab}\delta T_{a'b'}} = \overline{T_{ab}}\;\overline{T_{a'b'}}\left(\delta_{a,a'}\delta_{b,b'} + \frac{2}{3g}\left[\delta_{a,a'} + \delta_{b,b'}\right] + \frac{2}{15g^2}\right)$$

$$\overline{\delta g^2} = \cancel{\frac{g^2}{M^2}} + \cancel{\frac{4}{3}\frac{g}{M}} + \frac{2}{15} \qquad \frac{g}{M} \sim \frac{l_e}{L} \ll 1$$

Universal conductance fluctuations

Universal conductance fluctuations

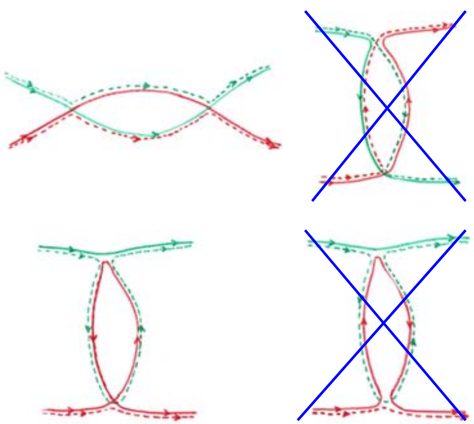


Conductance fluctuations = 2 conductances + 2 crossings

$$\overline{\delta G^2} = \left(\frac{e^2}{h}\right)^2 g^2 \times \frac{1}{g^2} \Rightarrow \left(\frac{e^2}{h}\right)^2$$

Universal conductance fluctuations and phase coherence

4 combinations



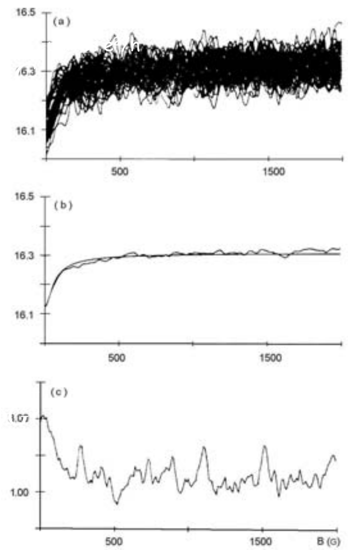
Diffuson « classical »

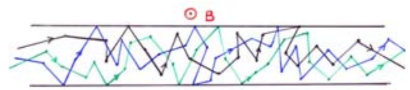
Cooperon « interference »

* A magnetic field suppresses
The contribution of the cooperon $P_{int}(t)$

$$\overline{\delta G^2} \to \frac{\overline{\delta G^2}}{2}$$

Magneto-fingerprints





each trace represents an interference pattern

Average $\overline{G} = G_{cl} + \Delta G$

The WL correction ΔG is suppressed by the magnetic field

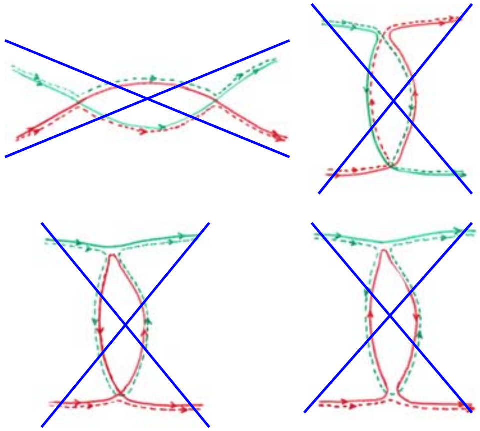
Variance $\overline{\delta G^2}$

In a field, the variance is reduced by a factor 2

D. Mailly, M. Sanquer, J. Physique I, 2, 357 (1992)

Universal conductance fluctuations and phase coherence

4 combinations



* Un champ magnétique supprime la contribution du cooperon $P_{int}(t)$

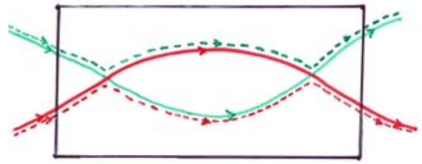
$$\overline{\delta G^2} \rightarrow \frac{\overline{\delta G^2}}{2}$$

* Dephasing processes destroys both

$$P_{int}(t) \quad \text{and} \quad P_{cl}(t)$$

$$L \gg L_\phi \quad \Rightarrow \quad \overline{\delta G^2} \rightarrow 0$$

Universal conductance fluctuations



Distribution of loops

$$\overline{\delta G^2} \sim \left(\frac{e^2}{h}\right)^2 g^2 \times \frac{1}{g^2} \times \langle t\, P(t) \rangle = \left(\frac{e^2}{h}\right)^2 g^2 \times \frac{1}{g^2} \times \int_{\tau_e}^{\min(\tau_D, \tau_\phi, \tau_B)} t\, P(t) \frac{dt}{\tau_D^2}$$

$$P(t) = P_{cl}(t) + P_{\text{int}}(t)$$

$$\overline{\delta G^2} = 12 \left(\frac{2e^2}{h}\right)^2 \int_{\tau_e}^{\tau_c} t\, P(t) \frac{dt}{\tau_D^2}$$

$$\tau_c = \min(\tau_D, \tau_\phi, \tau_B)$$

Example: the quasi-1D wire $L \ll L_\phi$ mesoscopic limit

← WL UCF →

$$\overline{\Delta g} = -2 \int_0^\infty P(t) \frac{dt}{\tau_D} \qquad \overline{\delta g^2} = 12 \int_0^\infty t\, P(t) \frac{dt}{\tau_D^2}$$

$$P(t) = \sum e^{-Dq^2 t}$$

$$\overline{\Delta g} = -2 \sum \frac{1}{(qL)^2} \qquad \overline{\delta g^2} = 12 \sum \frac{1}{(qL)^4}$$

$$q = \frac{n\pi}{L} \qquad n = 1, 2, \ldots, \infty$$

$$\overline{\Delta g} = -\frac{2}{\pi^2} \sum \frac{1}{n^2} \qquad \overline{\delta g^2} = \frac{12}{\pi^4} \sum \frac{1}{n^4}$$

$$\overline{\Delta g} = -\frac{1}{3} \qquad \overline{\delta g^2} = \frac{2}{15}$$

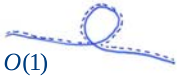
Example: the quasi-1D wire $L \gg L_\phi$ macroscopic limit

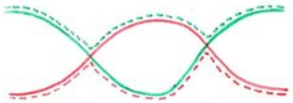
$$\overline{\delta g^2} = 12 \int_0^\infty t \left(\frac{\tau_D}{4\pi t}\right)^{d/2} e^{-t/\tau_\phi} \frac{dt}{\tau_D^2} \qquad P(t) = \left(\frac{\tau_D}{4\pi t}\right)^{d/2}$$

$$\overline{\delta g^2} \sim \left(\frac{\tau_\phi}{\tau_D}\right)^{\frac{4-d}{2}} = \left(\frac{L_\phi}{L}\right)^{4-d}$$

* classical addition of N incoherent elements : $N = \left(\frac{L}{L_c}\right)^d \longrightarrow \delta g \sim \frac{1}{L^{(4-d)/2}}$

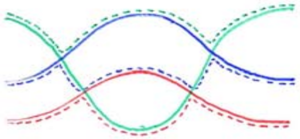
Gaussian fluctuations ?

$\overline{G} =$  $O(g)$ +  $O(1)$

$\overline{\delta G^2}^c =$ 

2 conductances and 2 crossings

$g^2 \times \frac{1}{g^2} \sim O(1)$

$\overline{\delta G^3}^c =$ 

3 conductances and 4 crossings

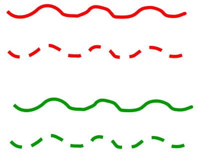
$g^3 \times \frac{1}{g^4} \sim O\left(\frac{1}{g}\right)$

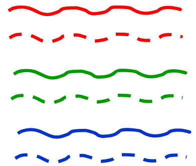
$\overline{\delta G^n}^c =$ n conductances and (2n- 2) crossings

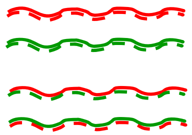
$$\overline{\delta G^n} \sim g^n \times \frac{1}{g^{2n-2}} \sim O\left(\frac{1}{g^{n-2}}\right)$$

Gaussian fluctuations in the limit $g \rightarrow \infty$

Rayleigh's distribution

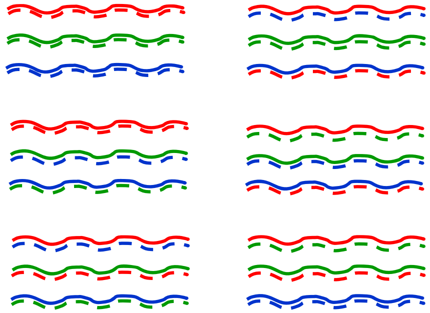






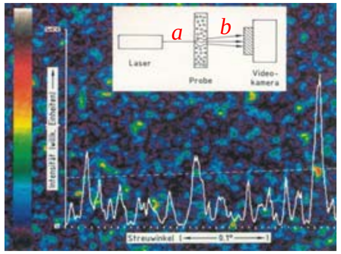
$$\overline{T_{ab}^2} = 2\,\overline{T}_{ab}^2$$

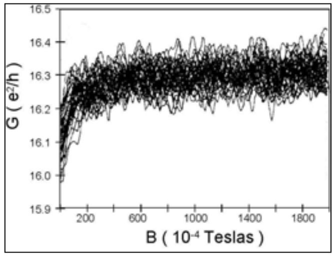
$$P(T_{ab}) = \frac{1}{\overline{T}} e^{-T_{ab}/\overline{T}}$$



$$\overline{T_{ab}^n} = n!\,\overline{T_{ab}}^n$$

Summary





$\delta T_{ab} = \overline{T_{ab}}$

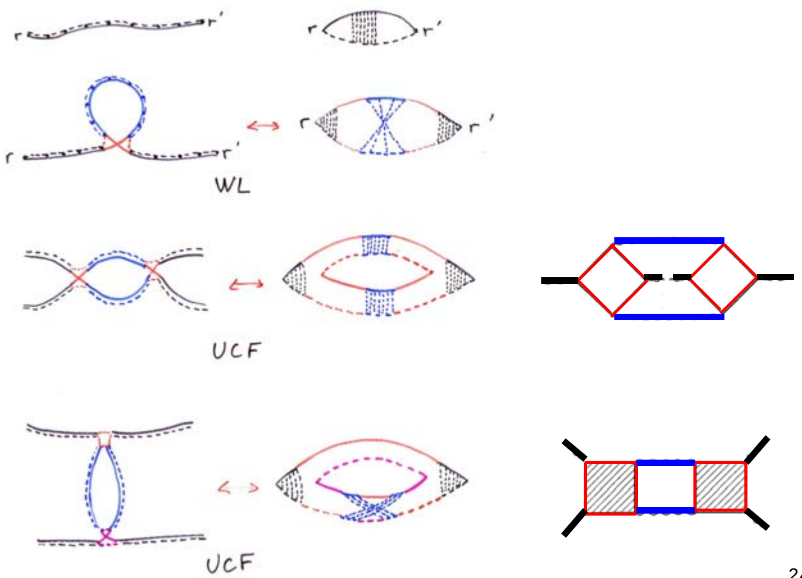
$\delta g \sim 1 \ll \overline{g}$

$$P(T_{ab}) = \frac{1}{\overline{T}} e^{-T_{ab}/\overline{T}}$$

$$P(g) = \frac{1}{\sqrt{2\pi\delta g^2}} e^{-\frac{(g-\overline{g})^2}{2\delta g^2}}$$

$$\delta T_{ab} = \sqrt{\overline{T_{ab}^2} - \overline{T_{ab}}^2}$$

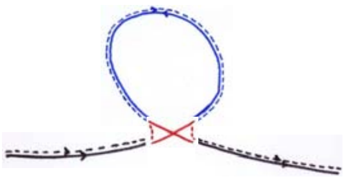
« Real » diagrams



Quantum Lego

Quantum transport of electrons and light in diffusive systems

« Lego » Classical diffusion (diffuson or cooperon)
Quantum crossings



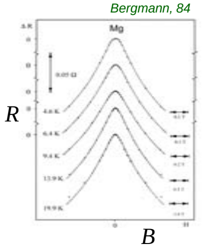
Simple formulation of phase coherent properties in the limit g>>1

Phase coherence length $L_\phi(T)$??

$L_\phi = \sqrt{D\tau_\phi}$

2-D

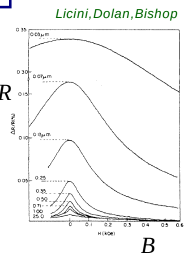
$f_{2D}\left(\frac{B}{B_*}\right)$ R



$B^* L_\phi^2 \sim \phi_0$ → $L_\phi \propto T^{-1/2}$

Quasi-1D

$f_{1D}\left(\frac{B}{B_*}\right)$ R

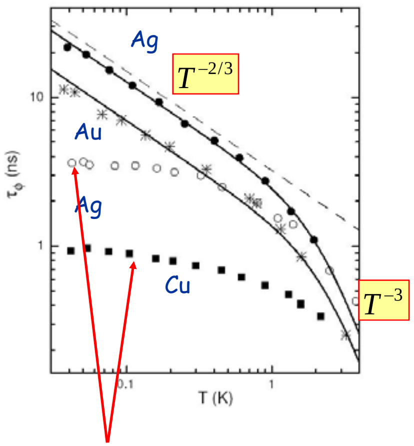


$B^* W L_\phi \sim \phi_0$ → $L_\phi \propto T^{-1/3}$

Phase coherence length $L_\phi(T)$??

$L_\phi = \sqrt{D\tau_\phi}$

Quasi-1D



Saturation due to magnetic impurities

e-e e-phonon

$$\frac{1}{\tau_\phi(T)} = AT^{2/3} + BT^3$$

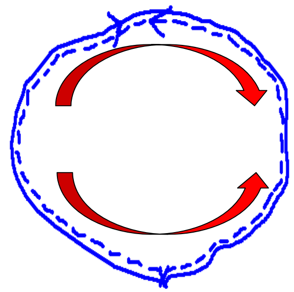
Saclay

Origin of phase decoherence : (very) qualitative

Random dephasing depends on the position of atoms (phonons), other electrons, magnetic impurities, etc.

Dephasing $4\pi\frac{\phi}{\phi_0} + \varphi(t) - \overline{\varphi}(t)$

$$e^{4i\pi\frac{\phi}{\phi_0} + i\Delta\varphi(t)}$$



$2\pi\frac{\phi}{\phi_0} + \varphi(t)$

$-2\pi\frac{\phi}{\phi_0} + \overline{\varphi}(t)$

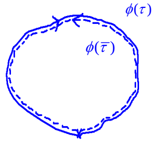
Averaging on the trajectories and the dynamics of external degrees of freedom :

$$\left\langle e^{i\Delta\varphi(t)}\right\rangle \sim e^{-\frac{1}{2}\left\langle \Delta\varphi^2(t)\right\rangle} \sim e^{-t/\tau_\phi}$$

Dephasing by electron-electron interaction

Weak-localization correction :

$$\Delta g = -4\int P_{\text{int}}(t)\ e^{-t/\tau_\phi}\ \frac{dt}{\tau_D}$$



$$\Delta g = -4\int P_{\text{int}}(t)\ \left\langle e^{i\Phi(t)}\right\rangle \frac{dt}{\tau_D}$$

quasi-1D wire

Dephasing : e-e interaction

Phase coherence time $\tau_\phi \propto T^{-2/3}$

*B.L. Altshuler, A.G. Aronov, D.E. Khmelnitskii, J. Phys. C **15**, 7367 (1982)*

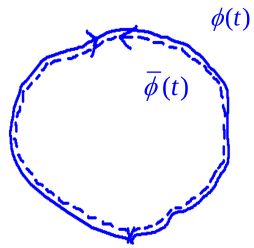
Dephasing by electron-electron interaction : qualitative

$\left\langle e^{i\Phi(t)}\right\rangle$?

e-e interaction = electric fluctuating potential

→ fluctuating phase

$$\phi(t) = \frac{e}{\hbar}\int_0^t V(r(\tau),\tau)\, d\tau \qquad \bar{\phi}(t) = \frac{e}{\hbar}\int_0^t V(r(\tau),\bar{\tau})\, d\tau$$



$$\left\langle e^{i\Phi(t)}\right\rangle \sim e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle} \qquad \Phi(t) = \phi(t) - \bar{\phi}(t) \qquad \bar{\tau} = t - \tau$$

$$\left\langle \Phi^2(t)\right\rangle = 2\frac{e^2}{\hbar^2}\int_0^t \left[\left\langle V(r_\tau,\tau)V(r_\tau,\tau)\right\rangle - \left\langle V(r_\tau,\tau)V(r_\tau,\bar{\tau})\right\rangle\right] d\tau$$

$$\frac{d\left\langle \Phi^2(t)\right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\left\langle V^2\right\rangle_t$$

Dephasing by electron-electron interaction : qualitative

$$\frac{d\left\langle \Phi^2(t)\right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\left\langle V^2\right\rangle_t \qquad \left\langle V^2\right\rangle_t \sim k_B T\, R_t \sim k_B T \frac{r_t}{\sigma_0 S}$$

Johnson-Nyquist noise

$$\frac{d\left\langle \Phi^2(t)\right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0 S} r_t$$

Diffusion $r_t \sim \sqrt{Dt}$

$$\left\langle \Phi^2(t)\right\rangle \sim \frac{k_B T\sqrt{D}}{\sigma_0 S}\, t^{3/2} \sim \left(\frac{t}{\tau_N}\right)^{3/2} \qquad \tau_N \sim \left(\frac{\hbar^2\sigma_0 S}{e^2 k_B T\sqrt{D}}\right)^{2/3} \propto \frac{1}{T^{2/3}}$$

« Nyquist time » (Aronov, Altshuler, Khmelnitskii)

$$\left\langle e^{i\Phi(t)}\right\rangle \sim e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle} \sim e^{-(t/\tau_N)^{3/2}}$$

Dephasing by electron-electron interaction : exact

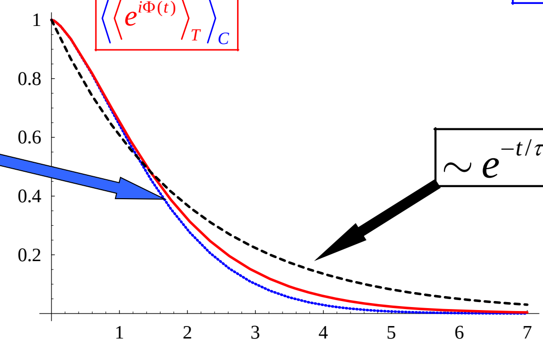
Thermal fluctuations — trajectories — approximation

$$\left\langle\left\langle e^{i\Phi(t)}\right\rangle_T\right\rangle_C = \left\langle e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle_T}\right\rangle_C$$

$$e^{-\frac{1}{2}\left\langle\left\langle \Phi^2(t)\right\rangle_T\right\rangle_C} = e^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2}}$$

$\left\langle\left\langle e^{i\Phi(t)}\right\rangle_T\right\rangle_C$

$e^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2}}$



$\sim e^{-t/\tau_\phi}$

$\tau_\phi \sim 2\tau_N$

*B.L. Altshuler, A.G. Aronov, D.E. Khmelnitskii, J. Phys. C **15**, 7367 (1982)* *G.M., E. Akkermans, Phys. Rev. Lett. **95**, 016403 (2005)*

Dephasing by electron-electron interaction : dimensionality effect

$$\frac{d\langle\Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\langle V^2\rangle_t$$

$$\langle V^2\rangle_t \sim k_B T\, R_t \sim k_B T \frac{r_t}{\sigma_0 S}$$ d=1

d=1

$$\frac{d\langle\Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0 S}\sqrt{Dt}$$

$$\langle\Phi^2(t)\rangle \sim \frac{k_B T\sqrt{D}}{\sigma_0 S}\, t^{3/2} \sim \left(\frac{t}{\tau_N}\right)^{3/2}$$

$$\tau_N \propto \frac{1}{T^{2/3}}$$

$$L_\phi \propto T^{-1/3}$$

d=2

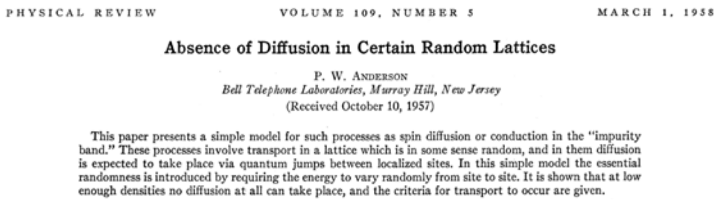
$$\frac{d\langle\Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0}$$

$$\langle\Phi^2(t)\rangle \sim \frac{k_B T}{\sigma_0}\, t \sim \frac{t}{\tau_N}$$

$$\tau_N \propto \frac{1}{T}$$

$$L_\phi \propto T^{-1/2}$$

Anderson localization


PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

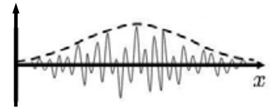
Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

In strong disorder, waves are localized by interference effects

Electrons, light, microwaves, sound, matter waves,



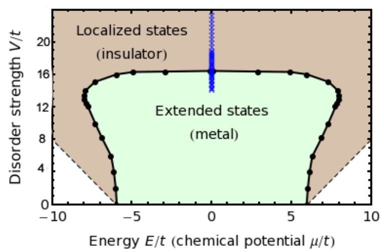
Fifty years of Anderson localization,
A. Lagendijk, B.v. Tiggelen and D.S. Wiersma, Physics Today, p.24 (Aug. 2009)

Anderson localization

$$\mathcal{H} = -t\sum_{\langle i,j\rangle} c_j^\dagger c_i + \sum_i W_i\, c_i^\dagger c_i$$

$$W_i \in [-W/2, W/2]$$

B.Kramer, A. MacKinnon, Rep. Prog. Phys. 56, 1469 (1993)



3D $W_c \simeq 16.5\,t$

$W < W_c$ *metallic regime*

$W > W_c$ *localized regime*

Near the transition…

$W < W_c$ $\sigma(W) \propto (W_c - W)^{\nu}$

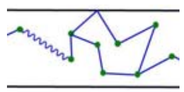
$W > W_c$ $\xi(W) \propto (W - W_c)^{-\nu}$

ν ?

1D, 2D no transition, no metallic regime, states exponentially localized

Metallic regime

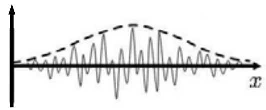
$\lambda_F \ll l_e$



Ohm's law $G = \sigma\frac{S}{L}$ $g(L) = \sigma L^{d-2}$

Localized regime

$\lambda_F \sim l_e$

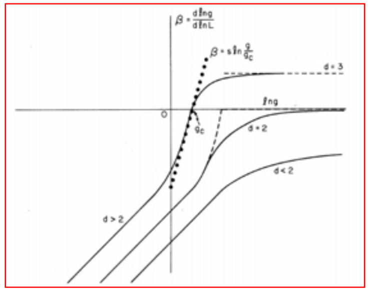


Quantum interference → states are exponentially localized in strong disorder

$g(L) = g_a e^{-L/\xi}$

d=1	all states are localized
d=2	marginal dimension
d=3	Anderson transition between metallic and localized regimes

Scaling theory : « gang of four »



Volume 42, Number 10 — PHYSICAL REVIEW LETTERS — 5 March 1979

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

E. Abrahams
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and

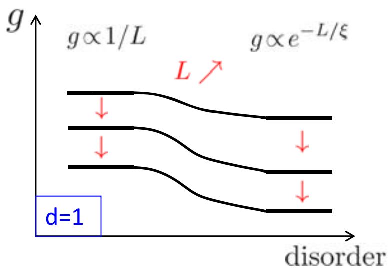
P. W. Anderson,[a] D. C. Licciardello, and T. V. Ramakrishnan[b]
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540

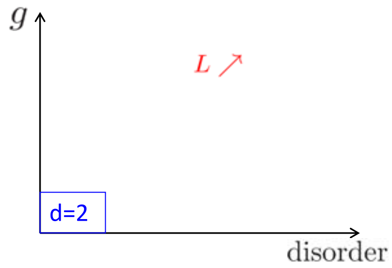
(Received 7 December 1978)

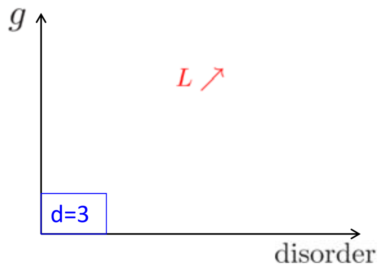
Arguments are presented that the $T=0$ conductance G of a disordered electronic system depends on its length scale L in a universal manner. Asymptotic forms are obtained for the scaling function $\beta(G) = d\ln G/d\ln L$, valid for both $G \ll G_c \approx e^2/\hbar$ and $G \gg G_c$. In three dimensions, G_c is an unstable fixed point. In two dimensions, there is no true metallic behavior; the conductance crosses over smoothly from logarithmic or slower to exponential decrease with L.

« gang of four »

metallic : $g = \sigma L^{d-2}$ localized : $g = g_a e^{-L/\xi}$







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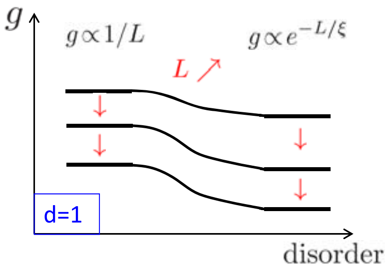

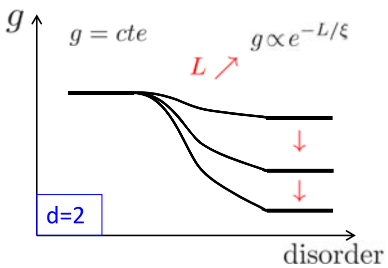

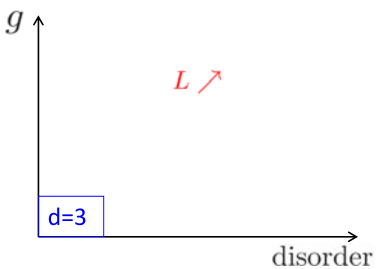

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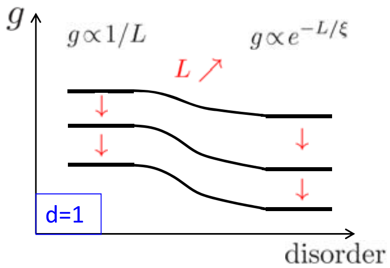
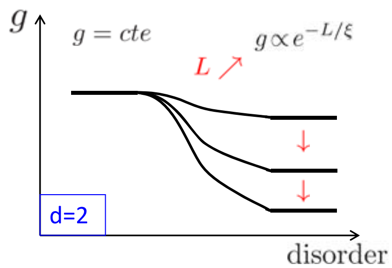
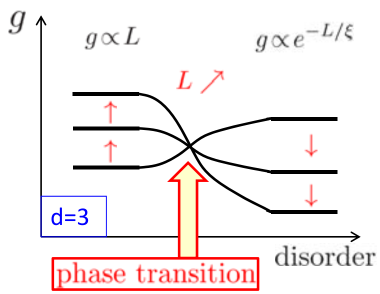






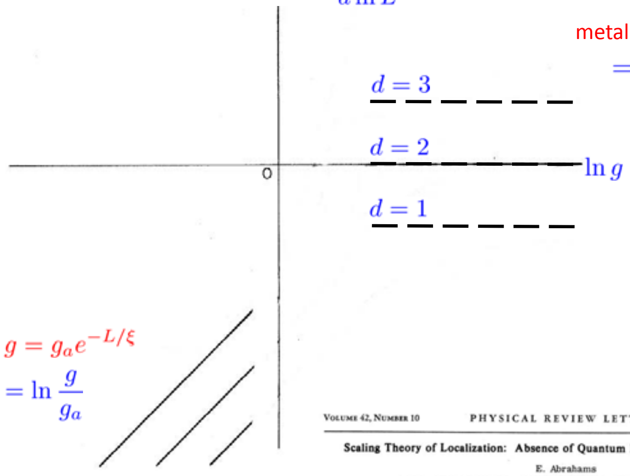
metallic : $g = \sigma L^{d-2}$

localized : $g = g_a e^{-L/\xi}$

$$\beta(g) = \frac{d \ln g}{d \ln L}$$

metallic : $g = \sigma L^{d-2}$

$\Longrightarrow \beta(g) = d - 2$



localized : $g = g_a e^{-L/\xi}$

$\Longrightarrow \beta(g) = \ln \dfrac{g}{g_a}$

VOLUME 42, NUMBER 10 PHYSICAL REVIEW LETTERS 5 MARCH 1979

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

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$\beta(g)$: first correction to the metallic regime

From weak localisation correction:

d=1 $\quad g = \dfrac{\sigma}{L} - c_1\left(1 - \dfrac{l_e}{L}\right)$

d=2 $\quad g = \sigma - c_2 \ln \dfrac{L}{l_e}$

d=3 $\quad g = \sigma L - c_3\left(\dfrac{L}{l_e} - 1\right)$

$\Longrightarrow \quad \beta(g) = \dfrac{\partial \ln g}{\partial \ln L} = d - 2 - \dfrac{c_d}{g}$

metallic : $g = \sigma L^{d-2}$

localized : $g = g_a e^{-L/\xi}$

$$\beta(g) = \frac{d \ln g}{d \ln L}$$

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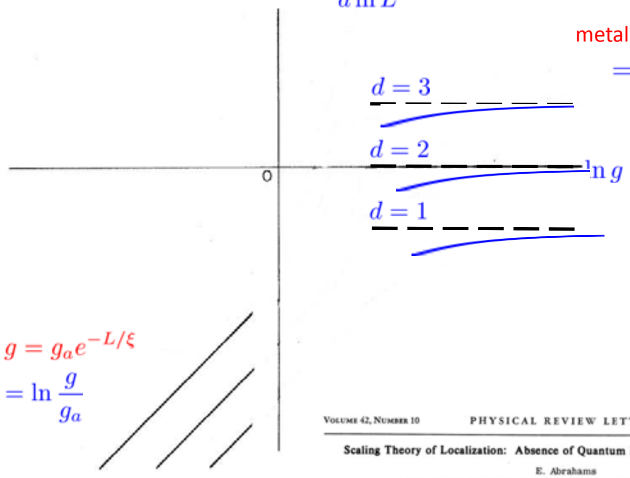

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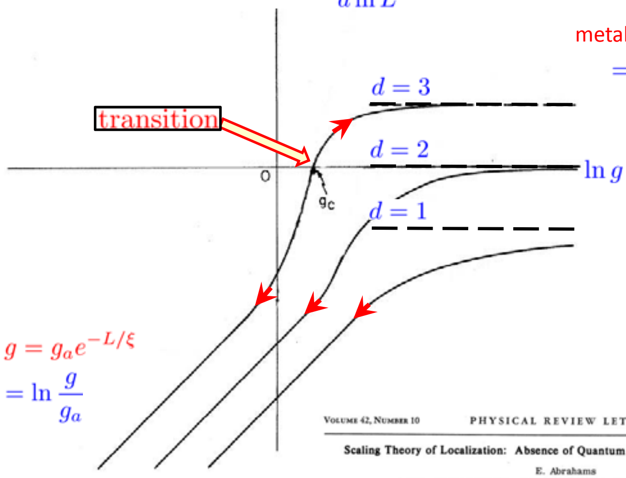

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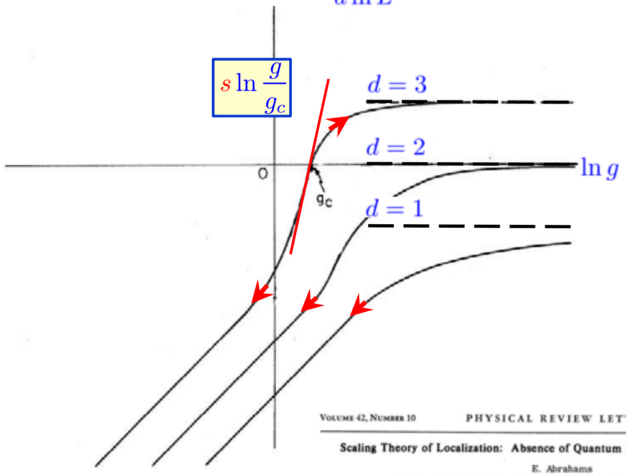
Scaling theory : « gang of four »

$$\beta(g) = \frac{d\ln g}{d\ln L}$$



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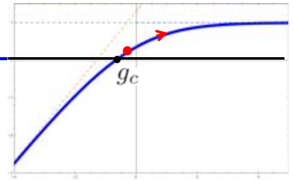


Start from a microscopic L_0 with g_0 ($\uparrow W$) near g_c ($\uparrow W_c$)

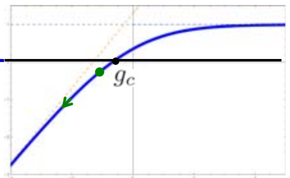
$$\beta(g) = \frac{d\ln g}{d\ln L} = s\ln\frac{g}{g_c} \qquad \frac{L}{L_0} = \left(\frac{\ln g_L/g_c}{\ln g_0/g_c}\right)^{1/s}$$

Metallic side $(g_0 > g_c)$

$$\sigma = \frac{g_L}{L} \propto \frac{g_c}{L_0}(\ln g_0/g_c)^{1/s} \quad \propto (g_0 - g_c)^{1/s}$$

$$\sigma \propto (W_c - W)^{1/s}$$

Scaling theory : « gang of four »



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$$\sigma \propto (W_c - W)^{1/s}$$

Insulating side $(g_0 < g_c)$

$$g_L = g_a \exp\left[-A(\ln|g_0/g_c|)^{1/s} L/L_0\right] = g_a e^{-L/\xi}$$

$$\xi \propto \frac{L_0}{(\ln|g_0/g_c|)^{1/s}} \quad \propto (g_c - g_0)^{-1/s}$$

$$\xi \propto (W - W_c)^{-1/s}$$

$$\nu = 1/s$$