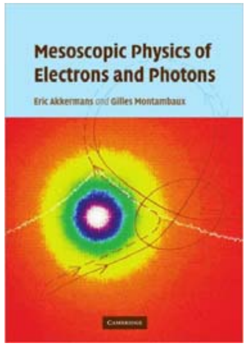
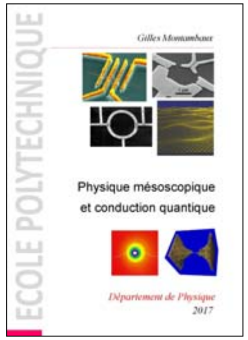


Quantum transport,
weak-localization, UCF, etc.



Introductions à Landauer-Büttiker, transport quantique, graphène : Poly en français accesssible sur

users.lps.u-psud.fr/montambaux

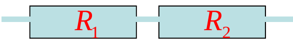
Publications on motion and merging of Dirac cones in graphene and artificial graphenes

Quantum transport = Mesoscopic physics = Phase coherence

Breakdown of classical laws of electronic transport

$$R \neq R_1 + R_2$$

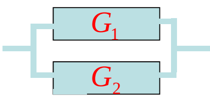
$$R \neq \rho \frac{L}{S}$$



$$G = \frac{1}{R}$$

$$G \neq G_1 + G_2$$

$$G \neq \sigma \frac{S}{L}$$



cf. Two path interferometer…

Outline

I - From classical transport to quantum transport

Disorder and phase coherence, the important length scales

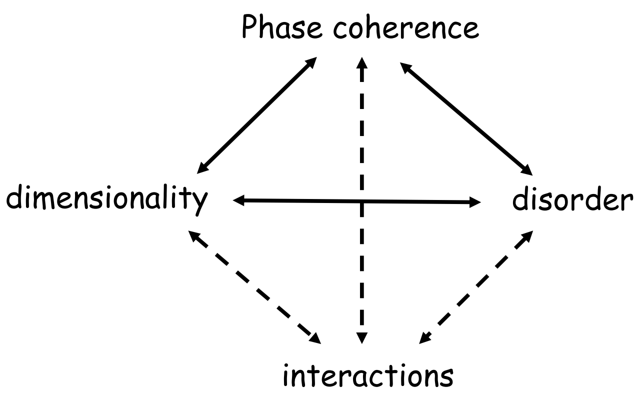
Different regimes of transport
- Ballistic classical (Sharvin)
- Ballistic quantum (quantization of conductance)
- Diffusive classical (Ohm-Drude)
- Diffusive quantum (weak-localization , UCF)

What is specific in 2D ? Disorder effect and weak-localization

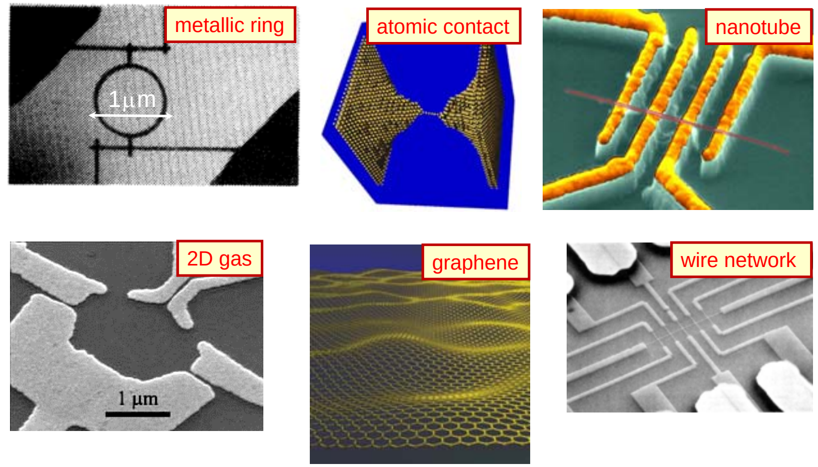
II - Landauer-Büttiker formalism of quantum transport
- Two terminal vs four terminal measurements
- Multiterminal formalism
- Application to QHE

III - Dirac matter, graphene and other materials (BN, bilayer, phosphorene)
- Engineering of Dirac points

The mesoscopic triangle



Quantum transport : what is conductance?



Landauer-Büttiker : conductance = transmission

Outline

I - From classical transport to quantum transport

Disorder and phase coherence, the important length scales

Different regimes of transport
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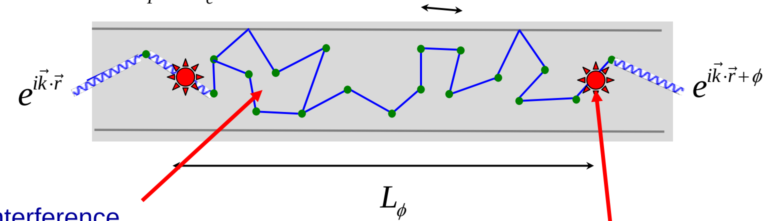
What is specific in 2D ? Disorder effect and weak-localization

Length scales

l_e Mean free path : distance between elastic collisions

$\lambda_F \ll l_e$ $\quad l_e = v_F \tau_e$ elastic collision time



interference

Elastic collisions do not break phase coherence

Interaction with an external degree of freedom (phonons, electrons, spin impurities… breaks phase coherence

$L_\phi(T)$ Phase coherence length $\quad L_\phi = \sqrt{D\tau_\varphi}$

Correspondance: time ←→ Length in the diffusive regime

phase coherence length $L_\phi^2 = D\,\tau_\phi$ phase coherence time

System size $L^2 = D\,\tau_D$

Thouless time

diffusion time

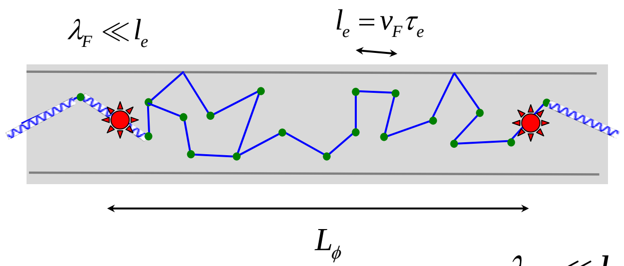
characteristic traversal time

More generally any (cut-off) time will be related to a characteristic length

$$L_c^2 = D\,\tau_c$$

Length scales

l_e Mean free path : distance between elastic collisions



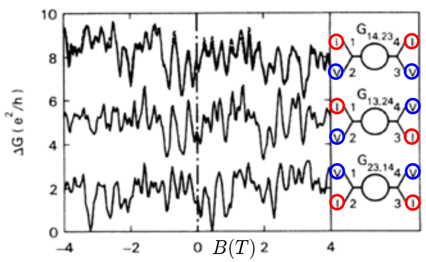
Weak-disorder (diffusive) mesoscopic regime $\lambda_F \ll l_e \ll L < L_\phi$

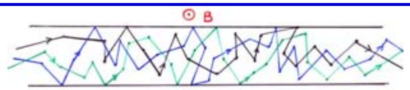
Ballistic regime $l_e \to \infty$

Strongly (Anderson) disordered regime $l_e < \lambda_F \ll L < L_\phi$

Reproducibles conductance fluctuations

$l_e \ll L < L_\phi$





~~$G = \frac{I}{V}$~~ → $G_{ij,kl} = \frac{I_{ij}}{V_{kl}}$

The conductance does not depend only on the system to be studied, but also to its connection to the external world

$\delta G \sim \frac{e^2}{h}$

One measures a conductance and not a conductivity

The Drude conductivity is an average property, valid if $L_\phi < L$

→ Beyond the average : interferences, fluctuations …

Disorder and phase coherence are independant notions

Disorder → complex interference pattern

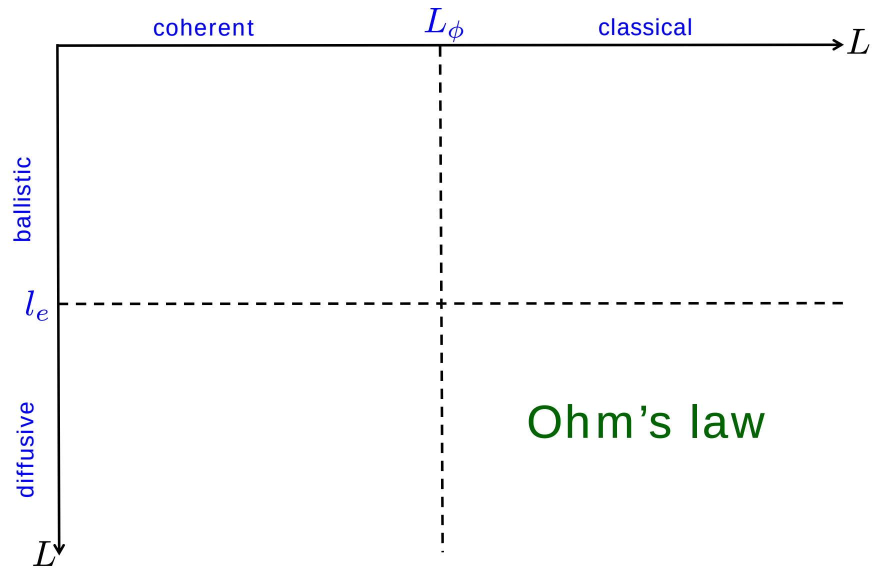
This interference pattern vanishes when phase coherence is lost

Phase coherence is limited by the coupling to other degrees of freedom :

phonons
magnetic impurities → L_ϕ
other electrons

Disorder does not kill the interference pattern, makes it more complex

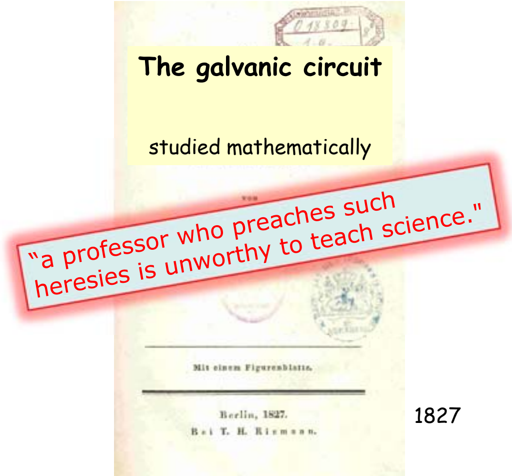
What about disorder average ?



$k_F l_e \gg 1$

Ohm's law

1789-1854



1827

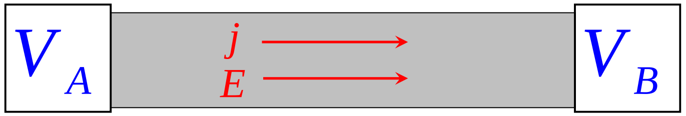
$$I = G V$$

$$G = \sigma \frac{S}{L}$$

G conductance, σ conductivity

$$G = 1/R$$

Ohm's law



$$\vec{j} = \sigma \vec{E}$$

$$I = G V$$

$$I = j S = \sigma E S$$

$$V = V_A - V_B = E L$$

$$G = \sigma \frac{S}{L}$$

G conductance, σ conductivity

Ohm's law

$$\sigma = \frac{n e^2 \tau}{m}$$ Drude-Sommerfeld formula
1900

$$G = \sigma \frac{S}{L}$$

$$\sigma = e^2 D \rho_0$$ Einstein formula

D diffusion coefficient

ρ_0 DOS at the Fermi level

Validity

Diffusive regime $L \gg l_e$

No quantum effects $L > L_\phi$

Brief reminder on density of states

$$\frac{1}{V}\sum_j \varphi(\epsilon_j) = \int \rho(\epsilon)\varphi(\epsilon)d\epsilon$$

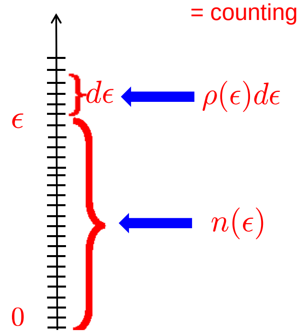
Number of states at a given energy : $\rho(\epsilon) = \frac{1}{V}\sum_j \delta(\epsilon - \epsilon_j)$

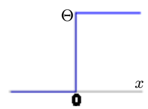
Number of states in an energy window $[\epsilon, \epsilon + d\epsilon]$: $\rho(\epsilon)d\epsilon$

It is very useful to define

= number of states of energy smaller than ϵ
= integrated DOS
= counting function

$$n(\epsilon) = \frac{1}{V}\sum_j \Theta(\epsilon - \epsilon_j)$$





$$\rho(\epsilon) = \frac{dn(\epsilon)}{d\epsilon}$$

by definition :
$n = n(\epsilon_F)$

Brief reminder on density of states

$$n(\epsilon) = \frac{1}{V}\sum_{\vec{k}} \Theta(\epsilon - \epsilon_{\vec{k}})$$

Continuum limit : $n(\epsilon) = \frac{1}{(2\pi)^d}\int_{\epsilon_{\vec{k}}<\epsilon} d^3\vec{k}$

$$n(\epsilon) = \frac{\text{Volume of } \vec{k} \text{ space } |\epsilon_{\vec{k}} < \epsilon}{(2\pi)^d}$$

If $\epsilon(\vec{k}) = \epsilon(|\vec{k}|)$

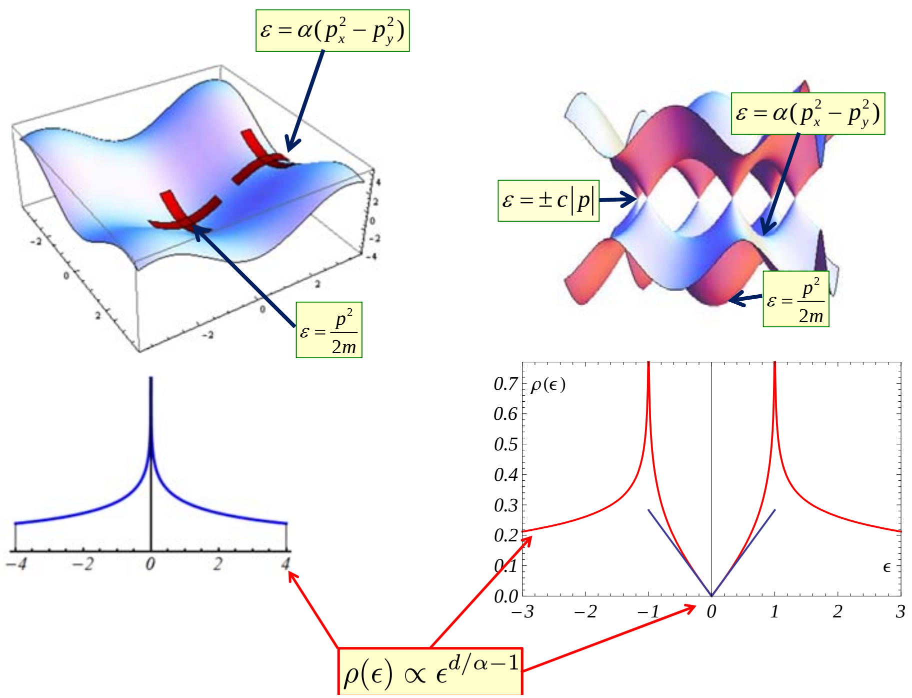
$$\rho(\epsilon) = \frac{dn}{d\epsilon}$$

$$n(\epsilon) = \frac{A_d}{(2\pi)^d}k(\epsilon)^d$$

$\epsilon(\vec{k}) \propto k^\alpha$

$$\rho(\epsilon) \propto \epsilon^{d/\alpha - 1}$$

$$\rho_0 = \rho(\epsilon_F) = \frac{dA_d}{\lambda_F^{d-1}hv_F}$$



What can we learn form classical transport ? back to Ohm's law

Ohm's law $G_{diff} = \sigma\frac{S}{L} = \sigma\frac{W^{d-1}}{L}$

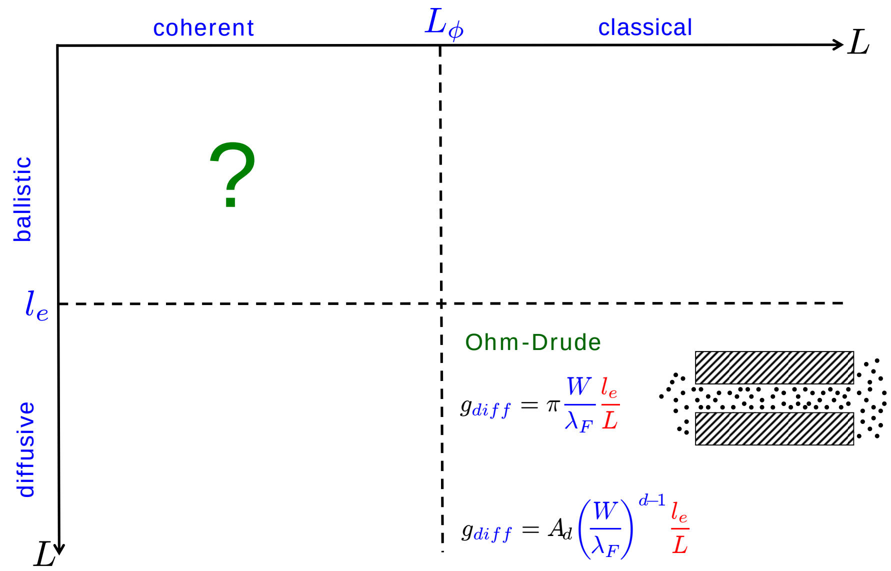
Conductivity $\sigma = e^2 D\rho_0$

Diffusion coefficient $D = \frac{v_F^2\tau_e}{d} = \frac{v_F l_e}{d}$

$$G_{diff} = e^2\rho_0 S\frac{v_F}{d}\frac{l_e}{L}$$

Density of states $\rho_0 = \rho(\epsilon_F) = \frac{2dA_d}{\lambda_F^{d-1}hv_F}$

$$G_{diff} = 2\frac{e^2}{h}A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$$



Ohm-Drude

$$g_{diff} = \pi \frac{W}{\lambda_F} \frac{l_e}{L}$$

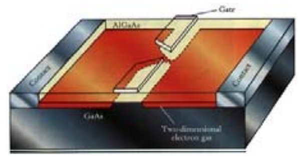
$$g_{diff} = A_d \left(\frac{W}{\lambda_F}\right)^{d-1} \frac{l_e}{L}$$

$k_F l_e \gg 1$

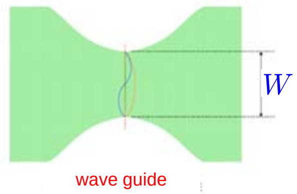
in units of $2\frac{e^2}{h}$

Conductance of a coherent ballistic system

$$G_q = 2\frac{e^2}{h} M = 2\frac{e^2}{h} \mathrm{Int} \frac{2W}{\lambda_F}$$

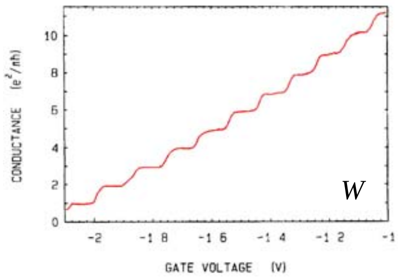


Quantum point contact **QPC**



wave guide

M transverse 'channels' 'modes'



Van Wees et al. PRL 1988; Wharam et al. J. Phys. ***C*** *1988*

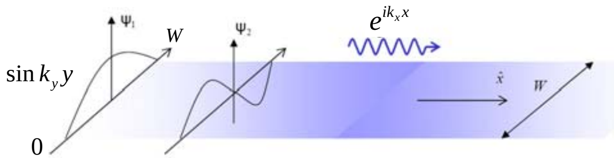
$2\frac{e^2}{h}$ per mode ...

see tomorrow's lecture on Landauer formula

Number of transverse channels

quantized modes

$$k_y = \frac{n_y \pi}{W}, \quad n_y > 0$$



since $k < k_F$ ⇒ $\frac{n_y \pi}{W} < k_F$

Number of transverse modes, channels

⇒ $M = \mathrm{Int}\left(\frac{k_F W}{\pi}\right) = \mathrm{Int}\left(\frac{2W}{\lambda_F}\right)$

Number of transverse channels

d=2 $\quad M = \mathrm{Int}\frac{k_F W}{\pi} = \mathrm{Int} 2\frac{W}{\lambda_F}$

d=3 $\quad M = \mathrm{Int}\frac{\pi}{4}\left(\frac{k_F W}{\pi}\right)^2 = \mathrm{Int}\,\pi\frac{W^2}{\lambda_F^2}$

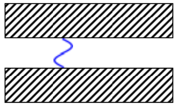
(d) $\quad M = \mathrm{Int}\frac{A_{d-1}}{2^{d-1}}\left(\frac{k_F W}{\pi}\right)^{d-1} = \mathrm{Int}\,A^{d-1}\frac{W^{d-1}}{\lambda_F^{d-1}}$

coherent L_ϕ classical L

ballistic

Quantized conductance

$g_q = \mathrm{Int}\, 2\frac{W}{\lambda_F}$



$g_q = \mathrm{Int}\, A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

?

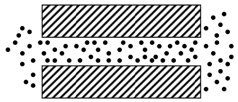
l_e

diffusive

M transverse 'channels' 'modes'

Ohm-Drude

$g_{diff} = \pi \frac{W}{\lambda_F}\frac{l_e}{L}$



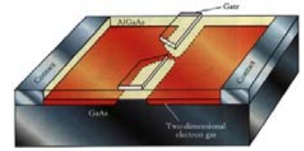
$g_{diff} = A_d \left(\frac{W}{\lambda_F}\right)^{d-1} \frac{l_e}{L}$

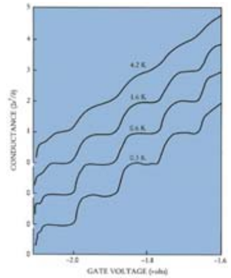
L

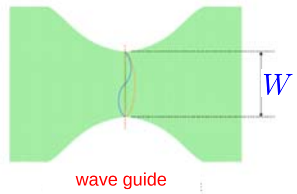
$k_F l_e \gg 1$

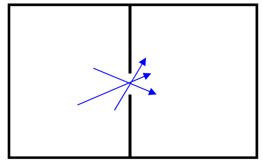
Conductance of a classical ballistic system

$$G_q = 2\frac{e^2}{h} M = 2\frac{e^2}{h}\mathrm{Int}\frac{2W}{\lambda_F}$$

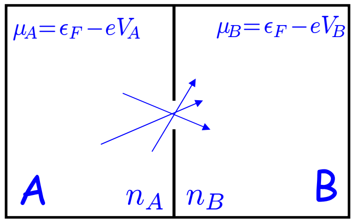








What do we get for classical particles (fermions) ?
Sharvin (1965)



$$S = W^{d-1}$$

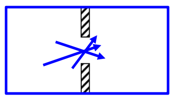
Number of particules of velocity v through the area S with incident angle θ during dt

→ Particle current $I_p = (n_A - n_B) S \langle v_x \rangle_+$

→ Electric current $I = (V_A - V_B) e^2 \rho S \langle v_x \rangle_+$

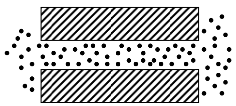
$$G_{bal} = e^2 \rho_0 \langle v_x \rangle_+ S$$

« Sharvin conductance »



$$G_{diff} = e^2 \rho_0 \frac{v_F}{d} S \frac{l_e}{L}$$

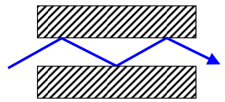
« Drude conductance »



Ballistic vs. diffusive incoherent

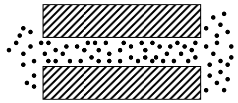
$$G_{bal} = e^2 \rho_0 \langle v_x \rangle_+ S$$

« Sharvin conductance »



$$G_{diff} = e^2 \rho_0 \frac{v_F}{d} S \frac{l_e}{L}$$

« Drude-Ohm conductance »



$$\rho_0 = \frac{2dA_d}{\lambda_F^{d-1} h v_F}$$

$$\langle v_x \rangle_+ = v_F \frac{A_{d-1}}{dA_d}$$

$$G_{bal} = 2\frac{e^2}{h} A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$$

Sharvin

$$G_{diff} = 2\frac{e^2}{h} A_d \left(\frac{W}{\lambda_F}\right)^{d-1} \frac{l_e}{L}$$

Drude

If ballistic coherent :

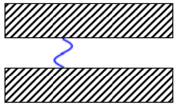
$$G_q = 2\frac{e^2}{h} A_{d-1} \mathrm{Int}\left(\frac{W}{\lambda_F}\right)^{d-1}$$

If diffusive coherent, quantum corrections
Weak-localization

coherent L_ϕ classical $\rightarrow L$

ballistic

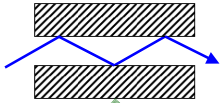
Quantized conductance



$g_q = \mathrm{Int}\, 2\frac{W}{\lambda_F}$

$g_q = \mathrm{Int}\, A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

Sharvin



$g_{bal} = 2\frac{W}{\lambda_F}$

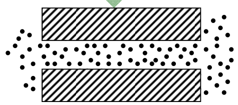
$g_{bal} = A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

l_e

diffusive

M transverse 'channels' 'modes'

Ohm-Drude



$g_{diff} = \pi\frac{W}{\lambda_F}\frac{l_e}{L}$

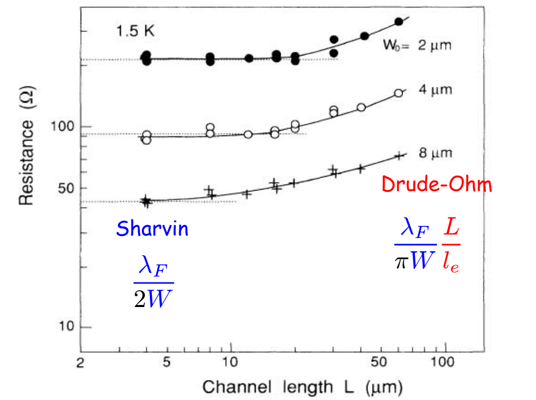
$g_{diff} = A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$

L

$k_F l_e \gg 1$

Ballistic $L < l_e$ $L > l_e$ Diffusive

in units $\frac{h}{2e^2}$



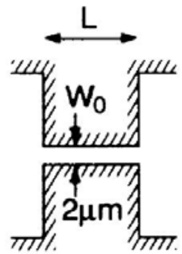


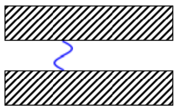
FIG. 2. Two-terminal resistances measured for the channels 2, 4, and 8 μm wide.

S. Tarucha et al., Sharvin resistance and its breakdown observed in long ballistic channels
Phys. Rev. B 47, 4064 (1993)

coherent L_ϕ classical $\rightarrow L$

ballistic

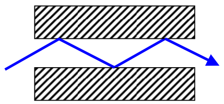
Quantized conductance



$g_q = \mathrm{Int}\, 2\frac{W}{\lambda_F}$

$g_q = \mathrm{Int}\, A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

Sharvin



$g_{bal} = 2\frac{W}{\lambda_F}$

$g_{bal} = A_{d-1}\left(\frac{W}{\lambda_F}\right)^{d-1}$

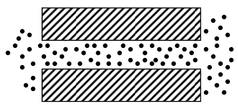
l_e

diffusive

Weak-localization corrections

$g = g_{diff} + \Delta g$

Ohm-Drude



$g_{diff} = \pi\frac{W}{\lambda_F}\frac{l_e}{L}$

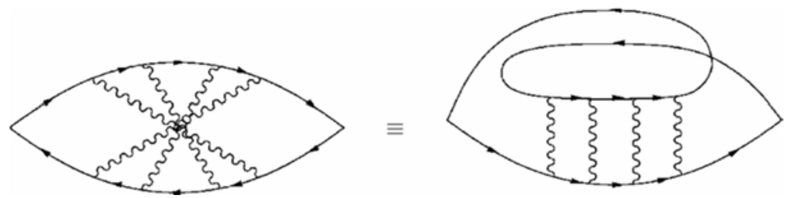
$g_{diff} = A_d\left(\frac{W}{\lambda_F}\right)^{d-1}\frac{l_e}{L}$

L

$k_F l_e \gg 1$

Quantum corrections in disordered media

Weak-localization

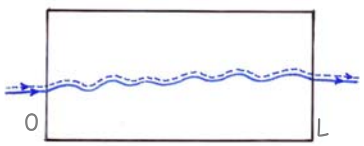


Conductance, transmission and probability

Conductance = transmission

Transmission through a disordered system= probability to cross the system

$$\overline{G} \propto P(0,L)$$



classical transport

Diffusion probability, microscopic approach

$P(r,r',t)$ probability to find a particule at *r'* , if it has been injected at *r*

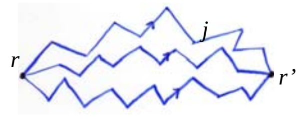
Quantum amplitude

Cf. Young's slits

$$G(r,r') = \sum_j A_j(r,r')$$

$$A_j(r,r') = |A_j(r,r')|\, e^{i\varphi_j(r,r')}$$

$$\varphi_j(r,r') = \frac{1}{\hbar}\int_r^{r'} p.dl$$



The probabiliy is the modulus square of the amplitude :

$$P(r,r') \sim \overline{|G(r,r')|^2} = \overline{\left|\sum_j A_j(r,r')\right|^2}$$

Disorder average

Diffusion probability, microscopic approach

Two contributions

Disorder average

$$P(r,r') = \overline{\sum_j |A_j(r,r')|^2} + \overline{\sum_{j\neq j'} A_j(r,r')A^*_{j'}(r,r')}$$

Classical term

Interference term → Quantum effects

Classical transport : only paired trajectories A_j A_j contribute
If the trajectories are different, the amplitudes A_j et $A_{j'}$ are different

⟹ uncorrelated phases
⟹ In average, the interference term disappears

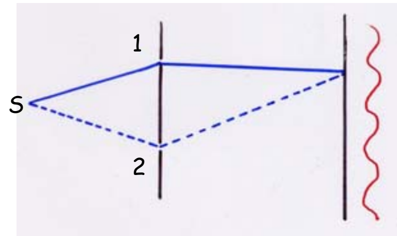
$P(r,r')$ =

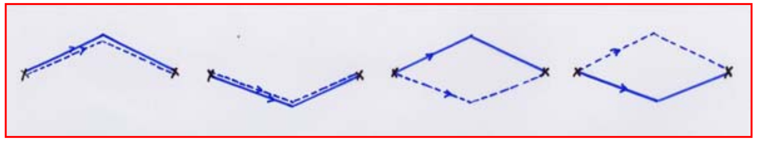
$P_{cl}(r,r')$

Example : Young slits

$$I = I_{cl} + I_{int}$$

$$I = |A_1 + A_2|^2$$

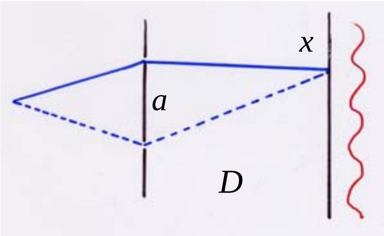




$$I = |A_1|^2 + |A_2|^2 + A_1 A_2^* + A_2 A_1^*$$

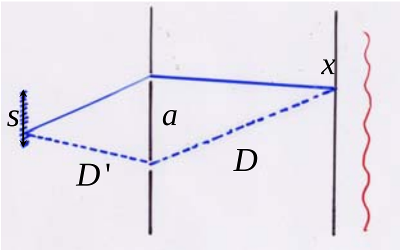
Example : Young slits

$$I = I_{cl} + I_{\text{int}}$$



$$I = I_{cl}\left(1+\cos\frac{kax}{D}\right)$$

$$I = I_{cl} + I_{\text{int}}$$



$$I = I_{cl}\left(1+\frac{\sin\frac{kas}{D'}}{\frac{kas}{D'}}\cos\frac{kax}{D}\right)$$

$$\langle I_{\text{int}}\rangle = 0$$

« disorder average »

Diffusion probability, microscopic approach

Two contributions

$$P(r,r') = \overline{\sum_j |A_j(r,r')|^2} + \overline{\sum_{j\neq j'} A_j(r,r')A^*_{j'}(r,r')}$$

Classical term

Interference term

→ Quantum effects

Classical transport : only paired trajectories A_j A_j contribute
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⟹ uncorrelated phases
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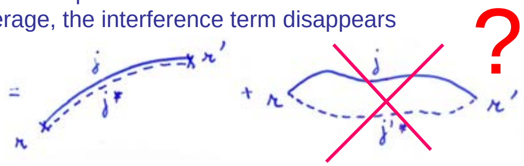
$P(r,r')$ =

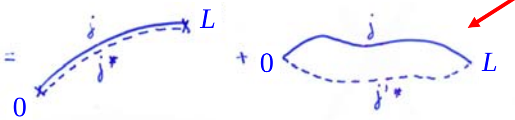


$P_{cl}(r,r')$

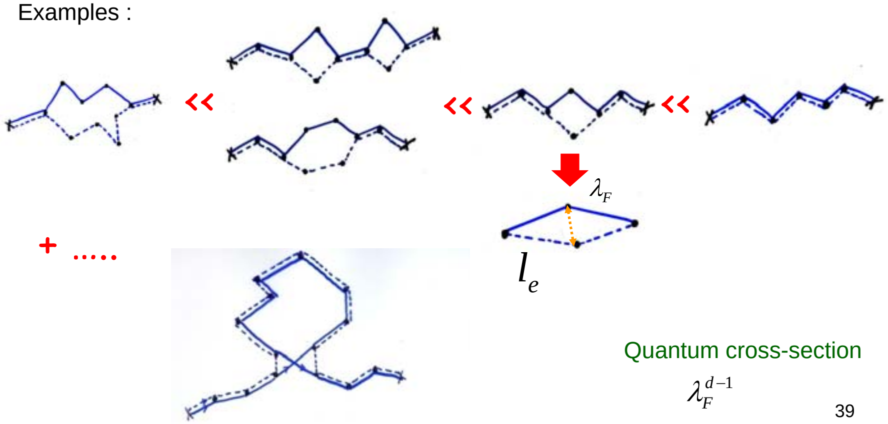
Quantum corrections

$$P(0,L) = \overline{\sum_j |A_j(0,L)|^2} + \overline{\sum_{j\neq j'} A_j(0,L)A^*_{j'}(0,L)}$$

$P(0,L)$ =



Examples :



λ_F

l_e

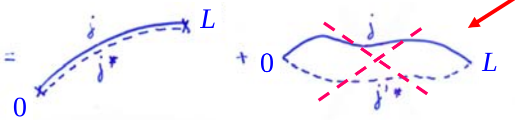
Quantum cross-section

λ_F^{d-1}

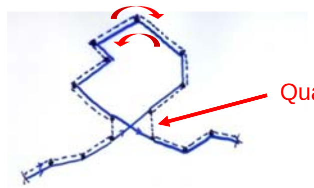
Quantum corrections

$$P(0,L) = \overline{\sum_j |A_j(0,L)|^2} + \overline{\sum_{j\neq j'} A_j(0,L)A^*_{j'}(0,L)}$$

$P(0,L)$ =



When performing disorder average, most contributions cancel, except when paired trajectories are very close to each other.
The remaining contribution corresponds to pairs of TR trajectories with a crossing.



Quantum crossing = Hikami box

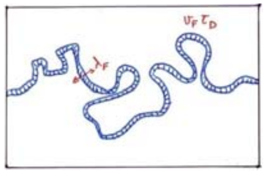
At the crossing of trajectories, there is dephasing. When ageraging over the positions of impurities, one can show that :

the relative amplitude of this contribution is of order of $\frac{1}{g}$ $\propto \lambda_F^{d-1}$

Evaluation of quantum corrections

The relative correction is proportionnal to the ratio :

$$\frac{\text{Volume of the trajectory explored during the diffusion through the sample}}{\text{Volume of the system}}$$



$$\frac{\Delta G}{G} = \frac{\Delta P}{P} \propto \frac{\lambda_F^{d-1} v_F \tau_D}{L^d} \sim \frac{1}{g} \quad !!!$$

Dimensionless conductance $G = g\,\frac{2e^2}{h}$

Coherent effects and quantum crossings

Quantum corrections are of relative order $\frac{1}{g}$

Classical transport $G_{cl} = g\,\frac{2e^2}{h}$

Quantum effects are of order $G_{cl} \times \frac{1}{g} \sim \boxed{\frac{e^2}{h}}$

Weak-localization corrections

Aharonov-Bohm (or Sharvin-Sharvin) oscillations

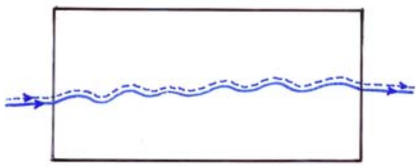
Universal conductance fluctuations

In a good metal ($g \gg 1$), quantum effects are small

Weak localization

Classical conductance

G_{cl}

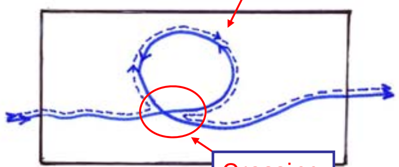


$\propto P_{cl}(0, L)$

Quantum correction ⇒ **One crossing** ⇒ **One loop**

Time reversed trajectories

$$\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle$$

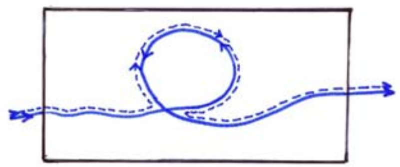


Crossing

$\propto \Delta P(0, L)$

$P_{\text{int}}(t)$ = distribution of number of loops with time t = return probabililty

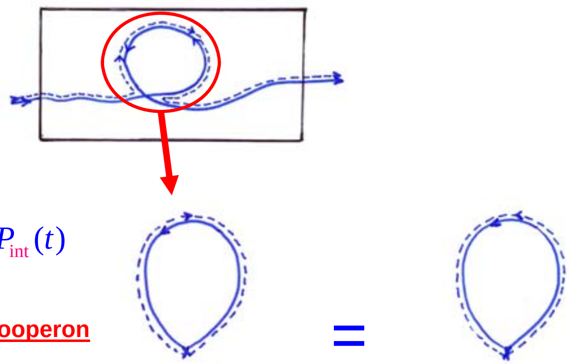
Weak-Localization



Loops and return probability
Weak-localisation in dimension d
Magnetic field, phase coherence

Weak localization : how to calculate $P_{\text{int}}(t)$?

$$\Delta G \sim -\frac{2e^2}{h}\langle P_{\text{int}}(t)\rangle$$



$P_{\text{int}}(t)$

Cooperon

Interference term

=

$P_{cl}(t)$

Diffuson

Classical return probability

$$P_{\text{int}}(r,r,t) = P_{cl}(r,r,t)$$

If time reversal invariance

Classical probability, diffusion equation

$P_{cl}(r,r')$ is solution of a classical diffusion equation:

$$\left(\frac{\partial}{\partial t} - D\Delta\right) P_{cl}(r,r',t) = \delta(r-r')\delta(t)$$

Solution in free space in d dimensions: $P_{cl}(R,t) = \frac{1}{(4\pi Dt)^{d/2}} e^{-\frac{R^2}{4Dt}}$

An important result, the return probability: $P(r,r,t) = \frac{1}{(4\pi Dt)^{d/2}}$

volume explored after time t

Weak localization : cut-offs

$$\Delta g \sim -\langle P_{\text{int}}(t)\rangle = -\int_{\tau_e}^{\tau_c} P_{\text{int}}(t)\frac{dt}{\tau_D} \qquad P(t) = \frac{V}{(4\pi Dt)^{d/2}}$$

Lower cut-off τ_e elastic collision time

Upper cut-off

time spent in the sample $\tau_D = \frac{L^2}{D}$

phase coherence time $\tau_\phi = \frac{L_\phi^2}{D}$

$$\tau_c = \min(\tau_D, \tau_\phi)$$

The return probability $P(t)$ increases for small d

Coherent effets are more important in low dimension

Weak localization : dependence on dimensionality

$$\Delta g \sim -\langle P_{\text{int}}(t)\rangle = -\int_{\tau_e}^{\tau_c} P_{\text{int}}(t)\frac{dt}{\tau_D} \qquad P(t) = \frac{V}{(4\pi Dt)^{d/2}}$$

$$\int_{\tau_e}^{\tau_\phi} \frac{dt}{t^{d/2}} \rightarrow$$

- $\sqrt{\tau_\phi} - \sqrt{\tau_e}$ $\quad d = 1 \quad (quasi-1D)$
- $\ln\frac{\tau_\phi}{\tau_e}$ $\quad d = 2$
- $\frac{1}{\sqrt{\tau_e}} - \frac{1}{\sqrt{\tau_\phi}}$ $\quad d = 3$

$$L_\phi^2 = D\tau_\phi$$

Weak localization : dependence on dimensionality

$d=1 \quad \Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L}$

$d=2 \quad \Delta g = -\frac{1}{\pi}\ln\frac{L_\phi(T)}{l_e} \qquad g = \frac{k_F l_e}{2}$

$d=3 \quad \Delta g = -\frac{1}{2\pi}\frac{L}{l_e} \qquad g = \frac{k_F^2 l_e L}{3\pi}$

Correction more important for small d
because return probability is enhanced

$g = A_d\left(\frac{k_F W}{2\pi}\right)^{d-1}\frac{l_e}{L} \quad \sim L^{d-2}$

Weak localization : dependence on dimensionality

$d=1 \quad \Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L} \qquad \frac{\Delta g}{g} \sim \frac{L_\phi}{M\, l_e}$

$d=2 \quad \Delta g = -\frac{1}{\pi}\ln\frac{L_\phi(T)}{l_e} \qquad g = \frac{k_F l_e}{2} \qquad \frac{\Delta g}{g} = \frac{2}{\pi}\frac{\ln(L_\phi / l_e)}{k_F l_e}$

$d=3 \quad \Delta g = -\frac{1}{2\pi}\frac{L}{l_e} \qquad g = \frac{k_F^2 l_e L}{3\pi} \qquad \frac{\Delta g}{g} \propto \frac{1}{k_F^2 l_e^{\,2}}$

$\frac{\Delta g}{g} \sim 1$ defines a new length scale at which perturbation breaks down

Localization length : $\xi_{1D} \sim M\, l_e$

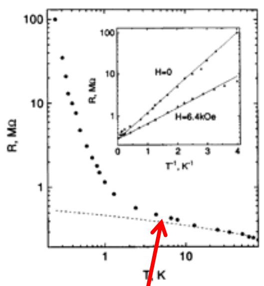
$\xi_{2D} \sim l_e e^{\frac{\pi}{2}k_F l_e}$

$g = A_d\left(\frac{k_F W}{2\pi}\right)^{d-1}\frac{l_e}{L} \quad \sim L^{d-2}$

$d=1 \quad (quasi-1D)$

$\Delta g = -\frac{L_\phi(T)}{L} \qquad g \sim M\frac{l_e}{L} \qquad \frac{\Delta g}{g} = -\frac{L_\phi}{M\, l_e}$

Localization length $\quad \xi = M\, l_e$



$M \sim 10 \qquad l_e \sim 20nm \qquad L_\phi \sim \xi$

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M.E. Gershenson et al.

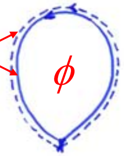
Crossover from Weak to Strong Localization in Quasi-One-Dimensional Conductors

The crossover from weak to strong localization in the resistance of quasi-1D conductors is observed for the first time with decreasing the temperature; it occurs when the phase-breaking length becomes comparable with the localization length. The signature of the strong-localization regime is an activation-type temperature dependence of the resistance and exponentially strong negative magnetoresistance. The magnetoresistance is well described by the theory of doubling of the localization length in quasi-1D conductors in strong fields; this provides a direct measurement of the localization length.

Phase coherence and magnetic field

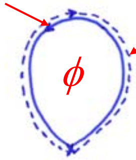
$P_{cl}(t)$ $\qquad$ $P_{\text{int}}(t)$

$2\pi\frac{\phi}{\phi_0}$



Diffuson

$2\pi\frac{\phi}{\phi_0} \qquad -2\pi\frac{\phi}{\phi_0}$



Cooperon

Cooperon: in a magnetic flux, paired trajectories get opposite phases

→ phase difference $4\pi\frac{\phi}{\phi_0}$

→ Oscillations of period $\frac{\phi_0}{2} = \frac{h}{2e}$

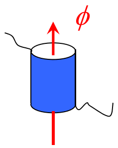
In a magnetic field, dephasing between time reversed trajectories

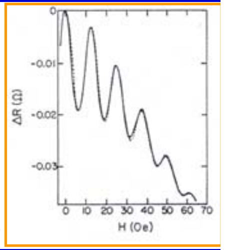
→The cooperon oscillates with flux

→ It cancels in a magnetic field

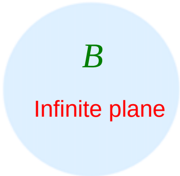
Effect of magnetic field (qualitative)

$$P_{int}(t) = P_{cl}(t)\, e^{4i\pi \frac{\phi}{\phi_0}}$$



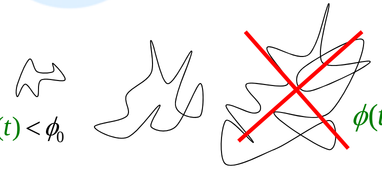


Sharvin-Sharvin



$$P_{int}(t) = P_{cl}(t) \left\langle e^{4i\pi \frac{\phi(t)}{\phi_0}} \right\rangle$$

Trajectories which enclose more than one flux quantum do not contribute to $P_{int}(t)$

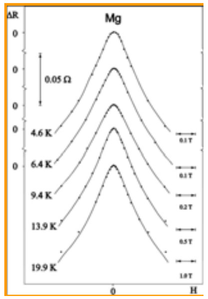


$\phi(t) < \phi_0$ $\phi(t) > \phi_0$

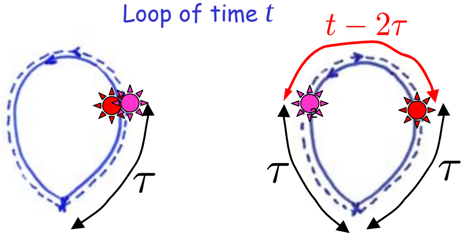
$$\sim e^{-t/\tau_B}$$

$$BD\tau_B = \phi_0$$

Bergman



Weak-localization = phase coherence



Phase coherence broken after a typical time τ_ϕ
Only trajectories of time $t < \tau_\phi$ contribute to the return probablity and to the WL

$$P_{int}(t) = P_{cl}(t)\, e^{-t/\tau_\phi}\, e^{4i\pi \frac{\phi}{\phi_0}}$$

Magnetic impurities, e-e interaction, magnetic impurities
Altshuler, Aronov, Khmelnitskii

Summary

$$\Delta g = -2 \int_0^\infty P^B_{int}(t) \left(e^{-t/\tau_\phi} - e^{-t/\tau_e} \right) \frac{dt}{\tau_D}$$

Contributions of closed diffusion trajectories whose size is limited by
Size of the system, phase coherence, magnetic field, etc.

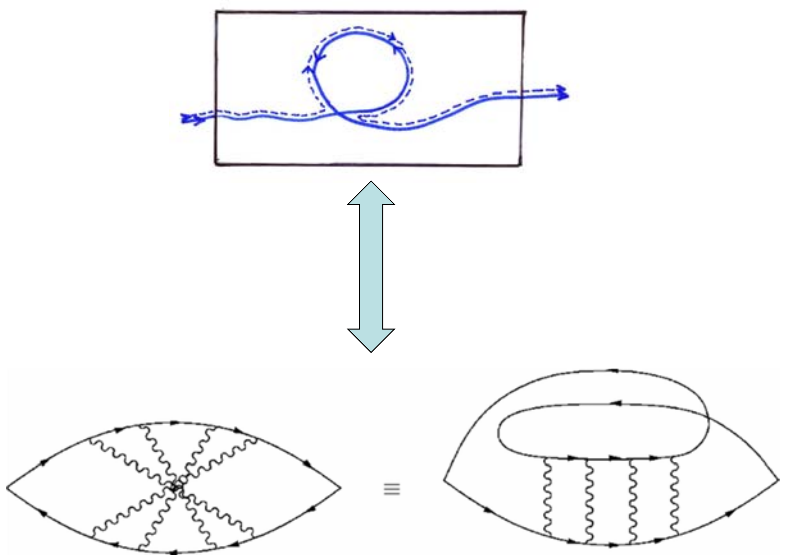
$$\Delta g \sim -2 \int_0^{\min(\tau_D, \tau_\phi, \tau_B)} \left(\frac{\tau_D}{4\pi t}\right)^{d/2} \frac{dt}{\tau_D}$$

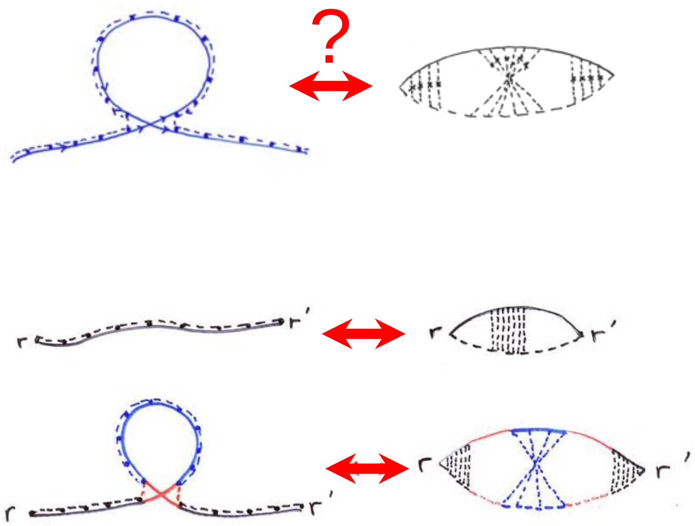
$$\tau_c \sim \min(\tau_D, \tau_\phi, \tau_B)$$

$$\Delta g = -\frac{L_c(T)}{L} \qquad d = 1 \quad (quasi-1D)$$

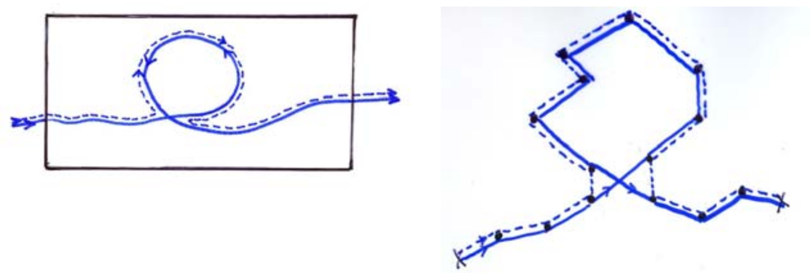
$$\Delta g = -\frac{1}{\pi} \ln \frac{L_c(T)}{l_e} \qquad d = 2$$

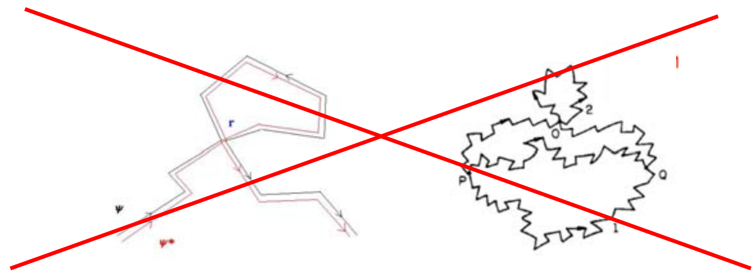
$$L_c = \sqrt{D\tau_c}$$





Quantum crossing is the reason why the WL correction is small



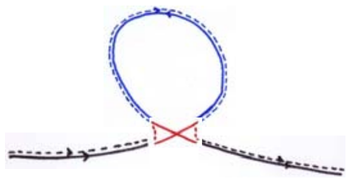


These representations are incorrect because they do not exhibit the crossing

Quantum Lego

Quantum transport of electrons and light in diffusive systems

« Lego » Classical diffusion (diffuson or cooperon)
Quantum crossings



Simple formulation of phase coherent properties in the limit g>>1