Quantum transport in 2D

Landauer-Büttiker formalism of quantum transport

GRAPHENE & CO, Cargèse April 2-13, 2018

Gilles Montambaux, Université Paris-Sud, Orsay, France

users.lps.u-psud.fr/montambaux

Quantum transport: what is conductance?

Landauer-Büttiker: conductance = transmission

Conductance = transmission

$G = \frac{2e^2}{h} T$

Landauer formula

$\frac{1}{V_1} \frac{1}{V_2}$

Landauer-Büttiker multiterminal formalism

R. Landauer (1927-1999)

M. Büttiker (1950-2013)

The 1D wire

Example: carbon nanotube

$V_1 \rightarrow$ scatterer

$\mu_1 = \mu - eV_1$

reservoir contact terminal

$V_2 \rightarrow$ lead

$\mu_2 = \mu - eV_2$

The electronic transport between the two reservoirs is a wave transmission through a potential barrier

M. Büttiker, Four terminal Phase-coherent conductor, PRL 1986
Hypotheses:

- A terminal absorbs electrons and injects them at a given potential and a given temperature.
- No phase relation between incoming and outgoing electrons in each terminal.
- The scatterer is elastic.
- The resistance of the reservoirs is negligible.

→ A problem of 1D quantum mechanics...

Current carried by an electron in a state $k$:

$$j = -\frac{eV}{L}$$

Summation over all filled states:

$$I = -\frac{2e}{h} \int_0^\infty T(\varepsilon) [f(\varepsilon + eV_1) - f(\varepsilon + eV_2)] d\varepsilon$$

(Remarkable result: the velocity has disappeared!)

Landauer formula

$$G = \frac{2e^2}{h} T(\varepsilon_F)$$

No scatterer (infinite conductivity?)

The conductance is finite and quantized!!!

$$G = \frac{2e^2}{h}$$
The perfect conductor has a finite and quantized conductance !!!

\[ G = \frac{2e^2}{h} \]

Where is the potential drop ?
Where is power dissipated ?
How to measure the conductance of the scatterer itself ?

Potential profile

Ballistic

No resistance in the sample
"contact" resistance
Power dissipated in the contacts

\[ R_c = \frac{V_1 - V_2}{I} = \frac{h}{4e^2} \]
\[ P = 2 \frac{e^2}{h} (V_1 - V_2)^2 \]

4 vs 2 terminals

Perfect sample: \( V_A = V_B \)

\[
I = 2 \frac{e^2}{h} (V_1 - V_2) \quad I = \infty \left(V_A - V_B\right)
\]

\[
G_2 = 2 \frac{e^2}{h} \quad G_4 = \infty
\]

With a scatterer \( V_A \neq V_B \)

\[
I = G_2 (V_1 - V_2) \quad I = G_4 (V_A - V_B)
\]

\[
G_2 = 2 \frac{e^2}{h} T 
\]

\[
G_4 = ???
\]

Potential profile

Ballistic

\[
G_i = 2 \frac{e^2}{h} T
\]

\[
G_i = \infty
\]

One scatterer

\[
G_i = 2 \frac{e^2}{h} T
\]

\[
G_i = 2 \frac{e^2}{h} \frac{T}{1-T}
\]

No dissipation in the wire

\[
G_i = 2 \frac{e^2}{h} T
\]

\[
G_i = 2 \frac{e^2}{h} \frac{T}{1-T}
\]

\[
G_4 = 2 \frac{e^2}{h} T
\]

\[
G_4 = \frac{e^2}{h} \frac{T}{R}
\]

\[
\text{« 2 terminal » conductance}
\]

\[
\text{« 4 terminal » conductance}
\]
The two-terminal resistance is the addition in series of the four-terminal resistance and the two contact resistances.

\[ V_A - V_B = R (V_1 - V_2) \]

\[ R_2 = \frac{V_1 - V_2}{I} = \frac{V_1 - V_A}{I} + \frac{V_A - V_B}{I} + \frac{V_B - V_2}{I} \]

\[ R_2 = R_c + R_d + R_c \]

**Landauer-Büttiker formulae**

Two-terminal conductance

\[ G_2 = 2 \frac{e^2}{h} T \]

Four-terminal conductance

\[ G_2 = 2 \frac{e^2}{h} \frac{\lambda_e}{\lambda_d} \]

**Sharvin resistance and its breakdown observed in long ballistic channels**

Sharvin resistance formula:

\[ R = \frac{\lambda_F}{2W} + \frac{\lambda_F}{\pi W l_e} \]

**Ohm-Drude**

\[ G = 2 \frac{e^2}{h} \frac{k_d W}{\lambda_d} \frac{\pi}{2} \frac{l_d}{L + \pi l_d / 2} \]

**Conductance = transmission**

\[ I = G V \]

Here, \( d = 2 \)

**Drude-Ohm**

\[ G = 2 \frac{e^2}{h} \frac{2W}{\lambda_d} \]

**Contact resistance in units**

\[ \frac{h}{2e^2} \]

**FIG. 2. Two-terminal resistances measured for the channels 2, 4, and 8 \( \mu \text{m} \) wide.**

The 2-terminal resistance is quantized

Multichannel Landauer formula

\[ \lambda = \frac{2W}{\sin k_y} \]

The current is the sum of the contribution of the different channels modes.
Multichannel Landauer formula

Current resulting from the transmission of a channel \( b \) to a channel \( a \)

\[
I_{ab} = \frac{2e^2}{h} T_{ab} (V_1 - V_2)
\]

Total current

\[
I = \frac{2e^2}{h} \sum_{a,b} T_{ab} (V_1 - V_2)
\]

Multichannel Landauer formula: clean wave guide

\[
G = \frac{2e^2}{h} \sum_{a,b} T_{ab} = \delta_{ab}
\]

\[
T_{ab} = \delta_{ab}
\]

\[
G = \frac{2e^2}{h} M = \frac{2e^2}{h} \text{Int}\frac{2W}{\lambda_F}
\]

Conductance of a coherent ballistic system

\( G_q = \frac{2e^2}{h} M = \frac{2e^2}{h} \text{Int}\frac{2W}{\lambda_F} \)

Quantum point contact (QPC)


\( \frac{2e^2}{h} \) per mode … see tomorrow’s lecture on Landauer formula

\( M \) transverse ‘channels’ ‘modes’
Conductance of a coherent ballistic system (finite $T$)

$$G_q = \frac{2e^2}{h} M = \frac{2e^2}{h} \text{Int} \frac{2W}{\lambda_F}$$

Quantum point contact (QPC)


$M$ transverse ‘channels’ ‘modes’

Wave guide


Wave guide

Conductance of a coherent ballistic system (finite $T$)

Quantization of the conductance : temperature effect

$$G = \frac{2e^2}{h} \text{Int} \frac{2W}{\lambda_F}$$

here:

$$G = \frac{2e^2}{h} \int M(\varepsilon) \left( -\frac{\partial f}{\partial \varepsilon} \right) d\varepsilon$$

$M(\varepsilon) = \sum \Theta(\varepsilon - \varepsilon_n)$

$$\varepsilon_n = n^2 \frac{\hbar^2 \pi^2}{2mW^2}$$

Characteristic energy:

$$\varepsilon^* = \frac{\hbar^2 \pi^2}{2mW^2} - 1K$$

$W \sim 250nm$

Landauer-Büttiker multiterminal formalism

$$G = \frac{2e^2}{h} T$$

Landauer formula

R. Landauer (1927-1999)

Landauer-Büttiker formulae

Two-terminal conductance

$$G = \frac{2e^2}{h} T$$

Four-terminal conductance

$$G = \frac{2e^2}{h} \frac{T}{R}$$

How many coefficients to characterize the « black box »?
**Landauer-Büttiker formulae**

Two-terminal conductance

\[ G_2 = \frac{2e^2}{h} T \]

Four-terminal conductance

\[ G_4 = \frac{2e^2}{h} T \]

In general depends on 9 transmission coefficients…

**Time Reversal Symmetry**

\[ T_{ij}(B) = T_{ji}(-B) \]

\[ B = 0 \quad T_{ij} = T_{ji} \]
Landauer-Büttiker formulae

Two-terminal conductance

\[ G_2 = \frac{2e^2}{hT} \]

Four-terminal conductance

\[ G_4 = \frac{2e^2}{h} \frac{D}{T_{31}T_{42} - T_{32}T_{41}} \]

In zero field, the \( 3 \times 3 \) submatrix is symmetric

4 terminal resistance in a carbon nanotube

Low T: the 4 terminal resistance can be negative

\[ R_{12,34} = \frac{h}{2e^2} \frac{T_{31}T_{42} - T_{32}T_{41}}{D} \]


Low T: the 4 terminal resistance can be negative, but power dissipate is positive

\[ \mathcal{P} = \frac{e^2}{2h} \sum_{i,j} (T_{ij} + T_{ji})(V_i - V_j)^2 \]
Symmetry of the two-terminal conductance

\[ T(B) = T(-B) \]

\[ G(B) = G(-B) \]


Symmetry of the four-terminal conductance

\[ G_{12,34} = \frac{I_{12}}{V_{34}} = 2\frac{e^2}{h} \frac{D}{T_{31}T_{42} - T_{32}T_{41}} \]

A. Benoit et al., Asymmetry in the magnetoconductance of metal wires and loops, Phys. Rev. Lett. 57, 1765 (1986)

Symmetry of the four-terminal conductance

\[ G_{14,23}(B) = G_{23,14}(-B) \]

Landauer-Büttiker formulae

In de Picciotto experiment, 6 coefficients reduce to one

\[ G_s = 2\frac{e^2}{h} \frac{D}{T_{31}T_{42} - T_{32}T_{41}} \]

\[ T_{31} = T_{32} = \varepsilon \]
\[ T_{34} = \varepsilon^2 \]
\[ T_{12} = (1 - \varepsilon)^2 \]

A. Benoit et al., Asymmetry in the magnetoconductance of metal wires and loops, Phys. Rev. Lett. 57, 1765 (1986)
Phase coherence

Non-locality

Quantum Hall effect

Bulk trajectories are pinned by disorder
Chiral edge trajectories propagate freely
Bulk insulator
Perfect « chiral » conductor at the edges

« Topological insulator »

Quantum Hall effect

Left-going and right-going electrons are spatially separated

Dissipation in the arrival terminal
Quantum Hall effect

Left-going and right-going electrons are spatially separated

Dissipation in the arrival terminal

This experiment shows that electrons stay at the chemical potential of the injection reservoir and exchange their energy at the arrival reservoir

Imaging of the dissipation in quantum Hall effect experiments