Quantum transport in 2D

Landauer-Büttiker formalism of quantum transport

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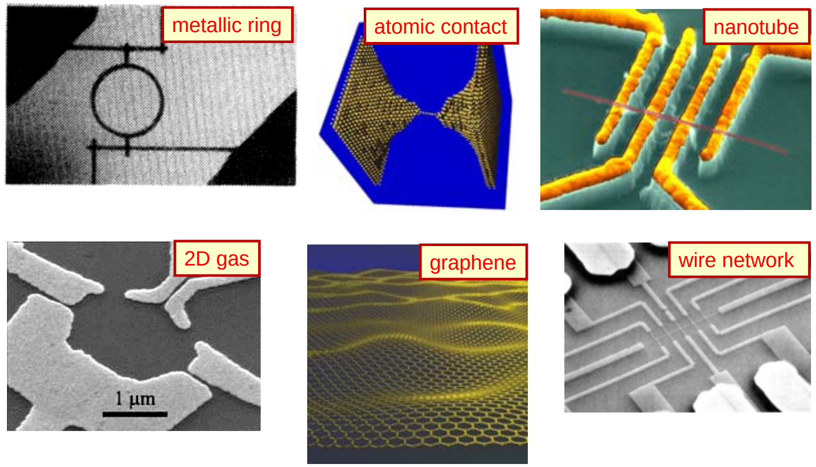
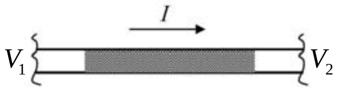
Quantum transport : what is conductance?



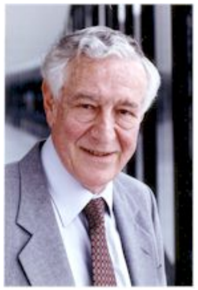
Landauer-Büttiker : conductance = transmission

Conductance = transmission

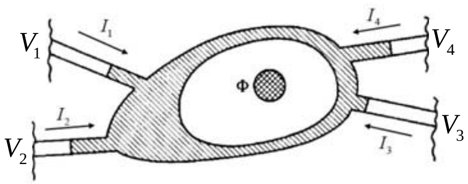


$$G = 2\frac{e^2}{h}T$$

Landauer formula



R. Landauer (1927-1999)



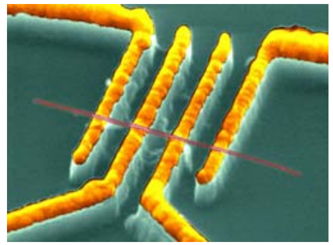
Landauer-Büttiker multiterminal formalism

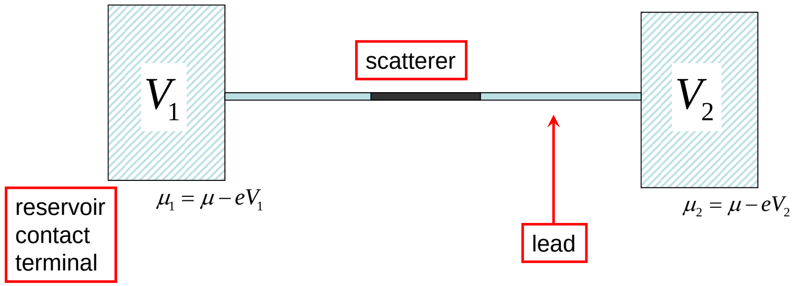
M. Büttiker (1950-2013)

M. Büttiker, Four terminal Phase-coherent conductor, PRL 1986

The 1D wire

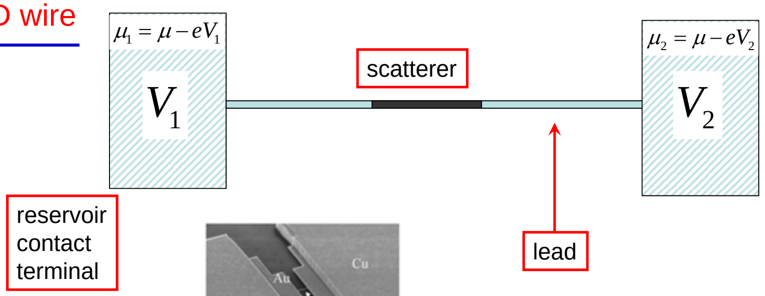
Example : carbon nanotube

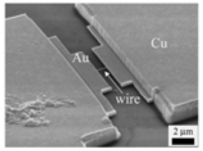




The electronic transport between the two reservoirs is a wave transmission through a potential barrier

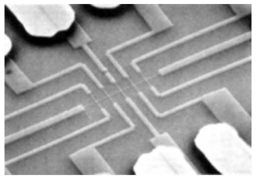
The 1D wire





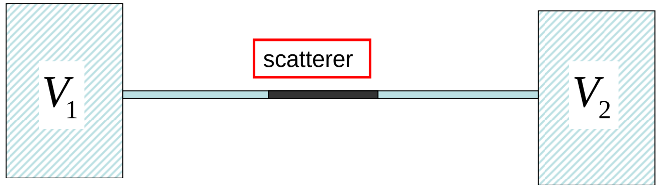
Hypotheses :

- A terminal absorbs electrons and inject them at a given potential and a given temperature.
- No phase relation between incoming and outgoing electrons in each terminal.
- the scatterer is elastic.
- The resistance of the reservoirs is negligible.



→ ***A problem of 1D quantum mechanics…***

The 1D wire



$\mu_1 = \mu - eV_1$ $\mu_2 = \mu - eV_2$

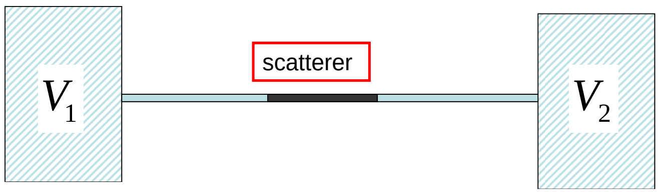
Current carried by an electron in a state k : $j = -\frac{ev_k}{L} T$

Summation over all filled states : $T(\varepsilon)$: transmission coefficient

$$I = -\frac{2e}{h}\int_0^\infty T(\varepsilon)[f(\varepsilon + eV_1) - f(\varepsilon + eV_2)]d\varepsilon$$

(Remarkable result : the velocity has disappeared !)

The 1D wire



$\mu_1 = \mu - eV_1$ $\mu_2 = \mu - eV_2$

Linear regime :

$$G = -\frac{2e^2}{h}\int_0^\infty T(\varepsilon)\frac{\partial f}{\partial \varepsilon}d\varepsilon$$

Low temperature :

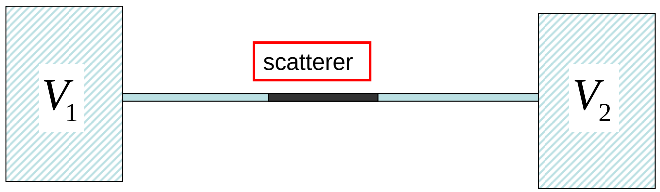
$$G = \frac{2e^2}{h}T(\varepsilon_F)$$

Conductance quantum :

$$\frac{e^2}{h} = 1/(25812{,}807\ \Omega)$$

(Remarkable result : the velocity has disappeared !)

The 1D wire



Landauer formula

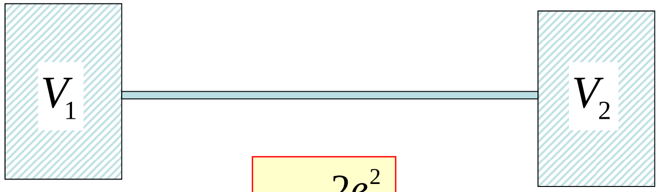
$$G = \frac{2e^2}{h}T(\varepsilon_F)$$

No scatterer (infinite conductivity ?)

$$G = \frac{2e^2}{h}$$

The conductance is finite and quantized !!!

The perfect conductor has a finite and quantized conductance !!!



$$G = \frac{2e^2}{h}$$

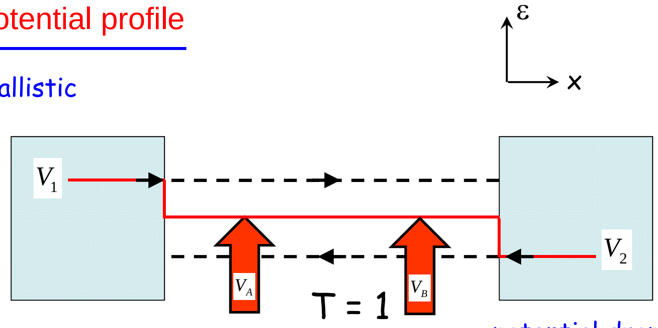
Where is the potential drop ?

Where is power dissipated ?

How to measure the conductance of the scatterer itself ?

Potential profile

Ballistic



$$G_2 = \frac{I}{V_1 - V_2} = 2\frac{e^2}{h}$$

$$G_4 = \frac{I}{V_A - V_B} = \infty$$

potential drop AT the contacts

No resistance in the sample

« contact » resistance

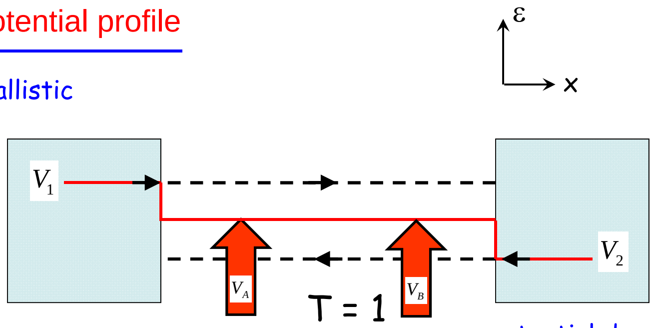
Power dissipated in the contacts

$$R_c = \frac{V_1 - V_m}{I} = \frac{h}{4e^2}$$

$$P = 2\frac{e^2}{h}(V_1 - V_2)^2$$

Potential profile

Ballistic



$$G_2 = 2\frac{e^2}{h}$$

$$G_4 = \infty$$

potential drop AT the contacts

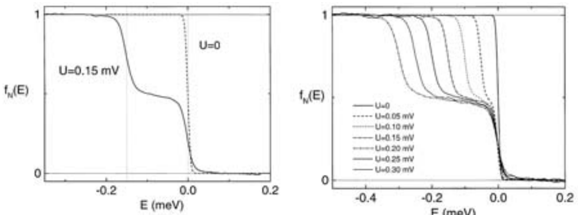


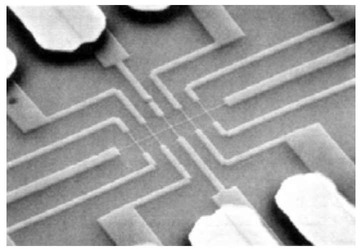
Fig. 6. Distribution function $f_N(E)$ at $x = L/2$, obtained from the deconvolution of the curves of Fig. 5 using (5). We have added dotted lines at $E = 0$ and $E = -0.15$ meV

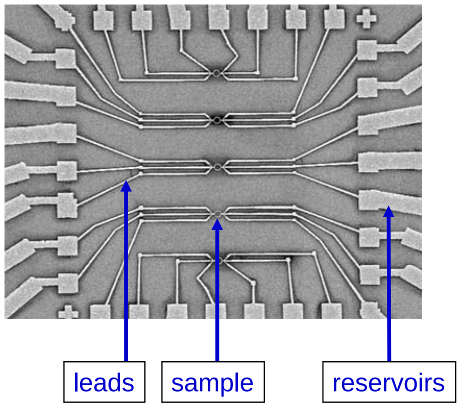
Fig. 7. Distribution function $f_N(E)$ at $x = L/2$, obtained from the measurement of the $dI/dV - V$ characteristics of the tunnel junction for U varying from 0 to 0.3 mV by steps of 0.05 mV

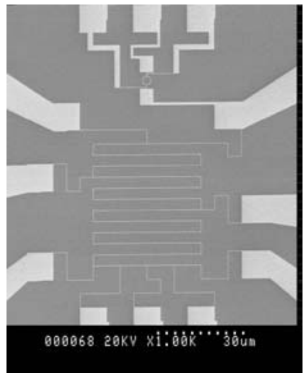
H. Pothier *et al., Energy distribution of electrons in an out-of-equilibrium metallic wire,* Phys. B **103**, 313-318 (1997)

« 2 terminal » resistance

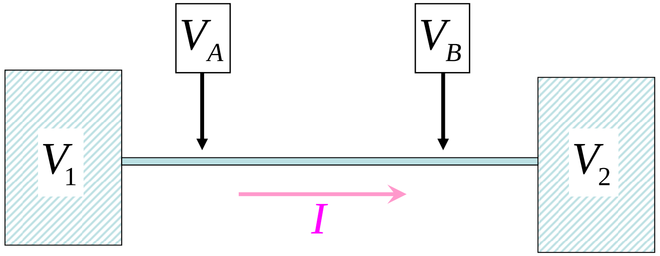
vs « 4 terminal » resistance







4 vs 2 terminals

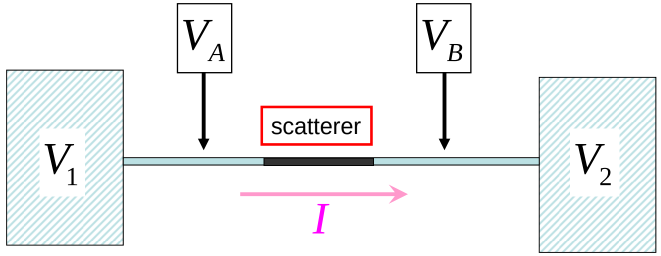


Perfect sample : $V_A = V_B$

$$I = 2\frac{e^2}{h}(V_1 - V_2) \qquad I = \infty\,(V_A - V_B)$$

$$\boxed{G_2 = 2\frac{e^2}{h}} \qquad \boxed{G_4 = \infty}$$

4 vs 2 terminals



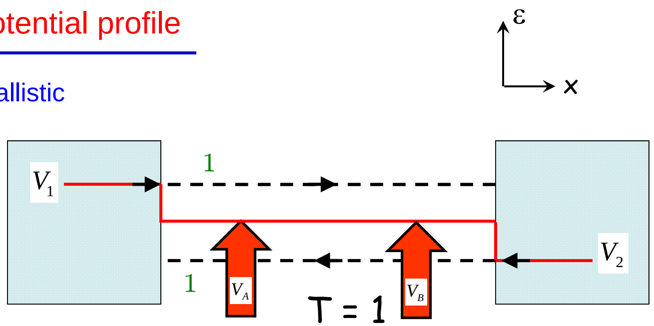
With a scatterer $V_A \neq V_B$

$$I = G_2(V_1 - V_2) \qquad I = G_4\,(V_A - V_B)$$

$$\boxed{G_2 = 2\frac{e^2}{h}T} \qquad \boxed{G_4 = ???}$$

Potential profile

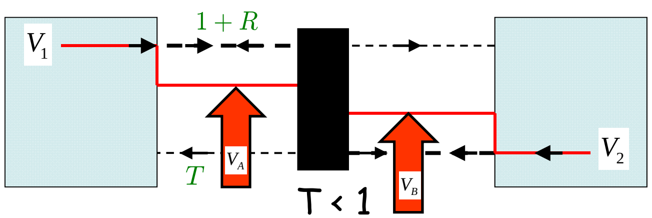
Ballistic



$$G_2 = 2\frac{e^2}{h}$$
$$G_4 = \infty$$

potential drop AT the contacts

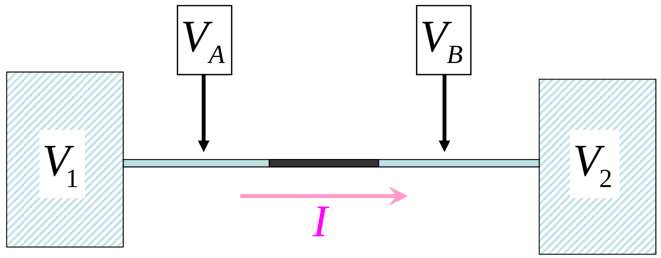
One scatterer



$$G_2 = 2\frac{e^2}{h}T$$
$$G_4 = 2\frac{e^2}{h}\frac{T}{1-T}$$

No dissipation in the wire

4 vs 2 terminals

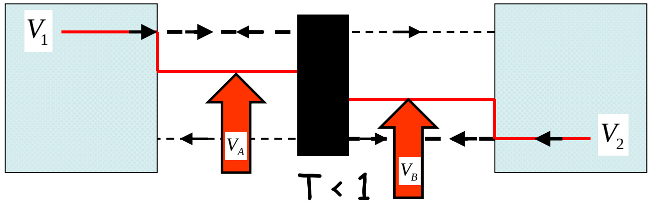


$$I = 2\frac{e^2}{h}T\,(V_1 - V_2) \qquad I = 2\frac{e^2}{h}\frac{T}{R}(V_A - V_B)$$

$$\boxed{G_2 = 2\frac{e^2}{h}T} \qquad \boxed{G_4 = 2\frac{e^2}{h}\frac{T}{R}}$$

« 2 terminal » conductance

« 4 terminal » conductance



$$V_A - V_B = R\,(V_1 - V_2)$$

$$G_2 = 2\frac{e^2}{h}T$$

$$R_2 = \frac{V_1 - V_2}{I} = \frac{V_1 - V_A}{I} + \frac{V_A - V_B}{I} + \frac{V_B - V_2}{I}$$

$$G_4 = 2\frac{e^2}{h}\frac{T}{R}$$

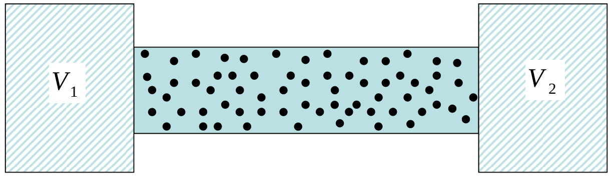
$$R_2 = R_c + R_4 + R_c$$

The two-terminal resistance is the addition in series
of the four-terminal resistance and the two contact resistances.

Conductance = transmission

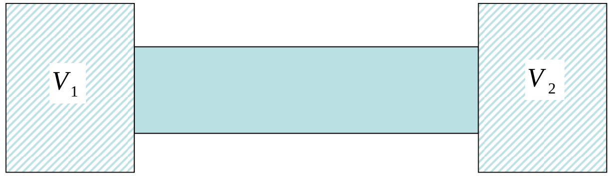
$I = G\,V$ conductance

here, d=2



$$G = 2\frac{e^2}{h}\frac{2W}{\lambda_F}\;\frac{\pi}{2}\frac{l_e}{L}$$

Ohm-Drude

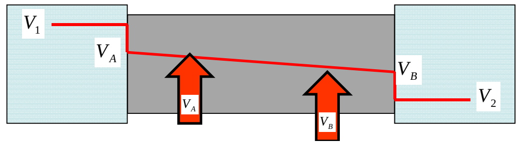


$$G = 2\frac{e^2}{h}\frac{2W}{\lambda_F}$$

Sharvin

$$G = 2\frac{e^2}{h}\frac{k_F W}{\pi}\;\frac{\pi}{2}\frac{l_e}{L + \pi l_e/2}$$

$$R_2 = R_c + R_4 + R_c$$



Ballistic $L < l_e$ $L > l_e$ Diffusive

in units $\frac{h}{2e^2}$

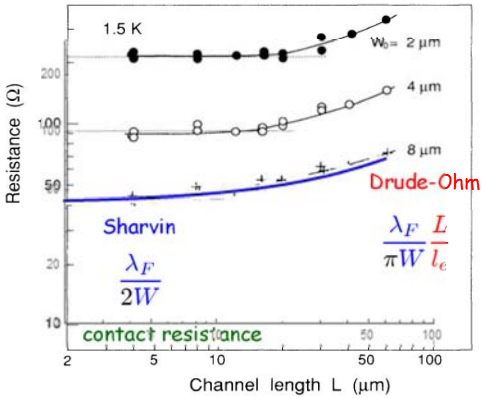
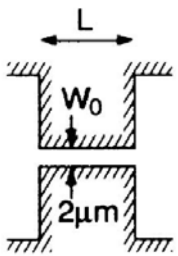


FIG. 2. Two-terminal resistances measured for the channels 2, 4, and 8 μm wide.

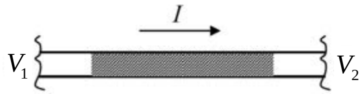


$$R = \frac{\lambda_F}{2W} + \frac{\lambda_F}{\pi W}\frac{L}{l_e}$$

S. Tarucha et al., Sharvin resistance and its breakdown observed in long ballistic channels Phys. Rev. B 47, 4064 (1993)

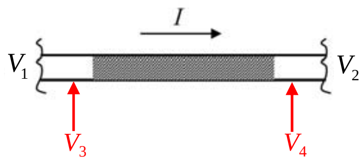
Landauer-Büttiker formulae

Two-terminal conductance



$$G_2 = 2\frac{e^2}{h}T$$

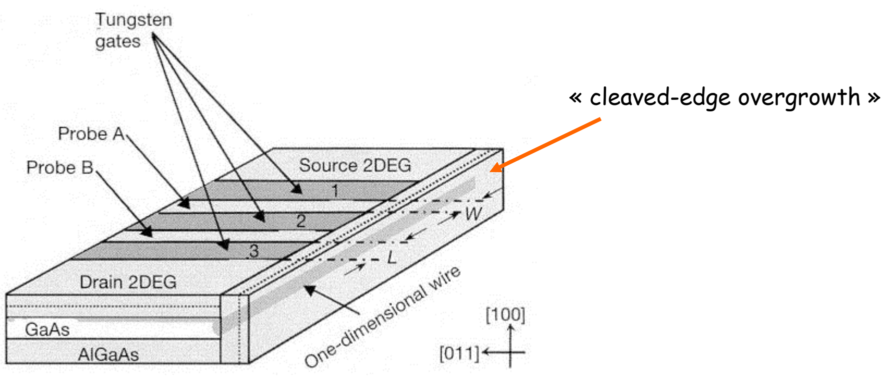
Four-terminal conductance



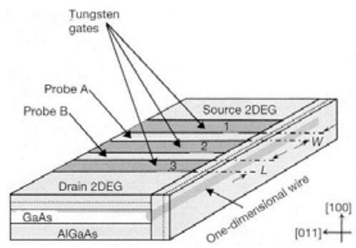
$$G_4 = 2\frac{e^2}{h}\frac{T}{R}$$

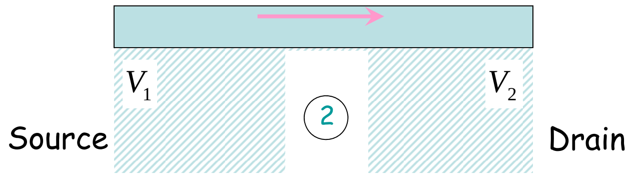
non-invasive voltage probes

Four terminal resistance of a ballistic quantum wire (2001)



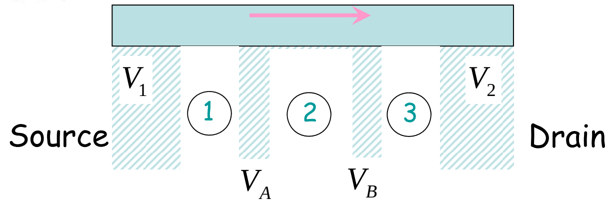
R. De Picciotto *et al.*, *Four terminal resistance of a ballistic quantum wire*, Nature 411, 51 (2001)

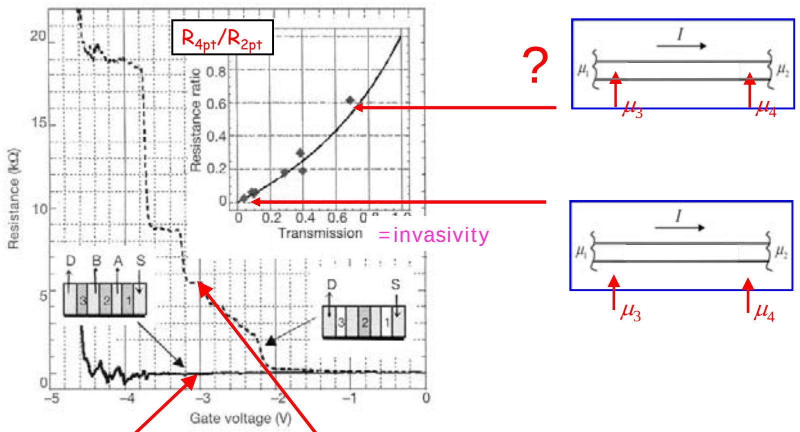




$$G_2 = 2\frac{e^2}{h}$$

$$G_4 = \infty$$





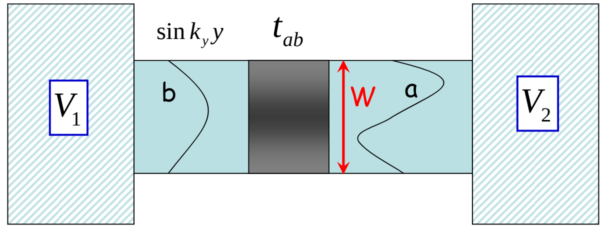
The 4-terminal resistance is 0

The 2-terminal resistance is quantized

For non invasive contacts

R. De Picciotto *et al.*, *Four terminal resistance of a ballistic quantum wire*, Nature 411, 51 (2001)

Multichannel Landauer formula

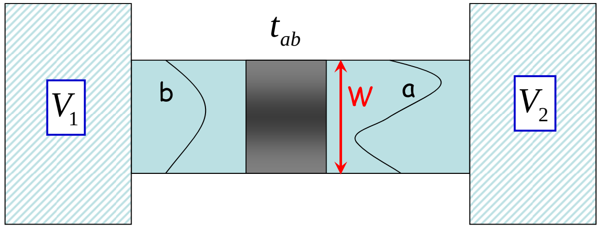


$$k_y = \frac{\pi}{W} \qquad k_y = \frac{2\pi}{W}$$

$$\lambda = 2W \qquad \lambda = W$$

The current is the sum of the contribution of the different | channels / modes

Multichannel Landauer formula



Current resulting from the transmission of a channel b to a channel a

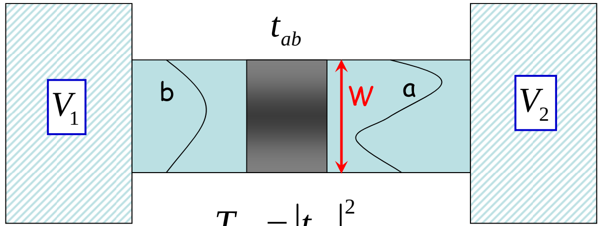
$$I_{ab} = \frac{2e^2}{h} T_{ab} (V_1 - V_2)$$

$$T_{ab} = |t_{ab}|^2$$

Total current

$$I = \frac{2e^2}{h} \sum_{a,b} T_{ab} (V_1 - V_2)$$

Multichannel Landauer formula

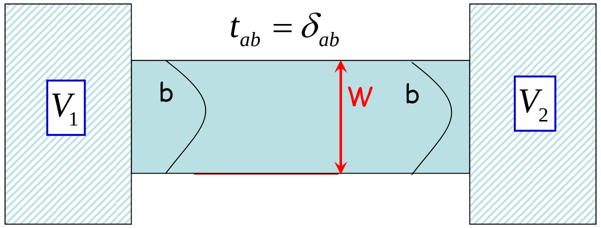


$$T_{ab} = |t_{ab}|^2$$

Multichannel Landauer formula

$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab}$$

Multichannel Landauer formula : clean wave guide



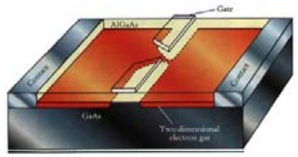
$$T_{ab} = \delta_{ab}$$

$$G = \frac{2e^2}{h} \sum_{a,b} T_{ab} \quad \Rightarrow \quad G = \frac{2e^2}{h} M$$

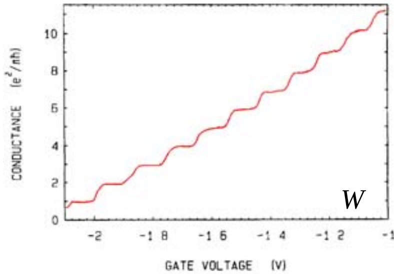
M is the number of transverse channels

Conductance of a coherent ballistic system

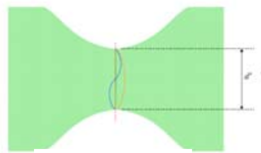
$$G_q = 2\frac{e^2}{h} M = 2\frac{e^2}{h} \mathrm{Int} \frac{2W}{\lambda_F}$$



Quantum point contact **QPC**



*Van Wees et al. PRL 1988; Wharam et al. J. Phys. **C** 1988*



wave guide

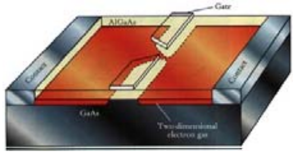
M transverse 'channels' 'modes'

$2\frac{e^2}{h}$ per mode ...

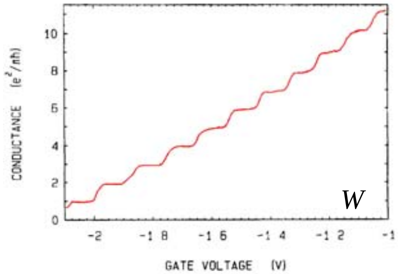
~~see tomorrow's lecture on Landauer formula~~

Conductance of a coherent ballistic system (finite T)

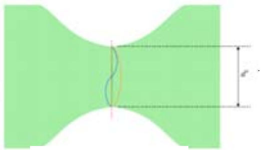
$$G_q = 2\frac{e^2}{h}M = 2\frac{e^2}{h}\mathrm{Int}\frac{2W}{\lambda_F}$$



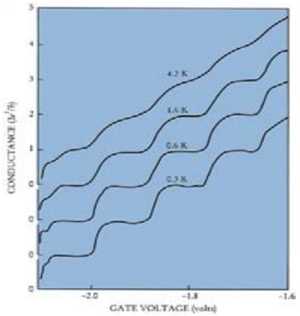
Quantum point contact **QPC**



*Van Wees et al. PRL 1988; Wharam et al. J. Phys. **C** 1988*



wave guide



M transverse 'channels' 'modes'

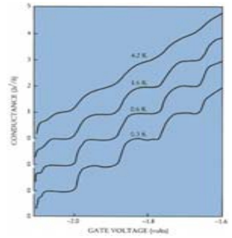
Quantization of the conductance : temperature effect

$$G = \frac{2e^2}{h}\sum_{a,b}\int T_{ab}(\varepsilon)\left(-\frac{\partial f}{\partial \varepsilon}\right)d\varepsilon$$

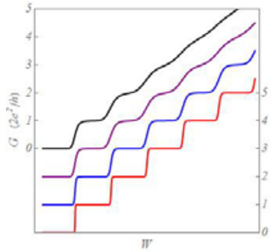
here :

$$G = \frac{2e^2}{h}\int M(\varepsilon)\left(-\frac{\partial f}{\partial \varepsilon}\right)d\varepsilon$$

$$M(\varepsilon) = \sum \Theta(\varepsilon - \varepsilon_n)$$



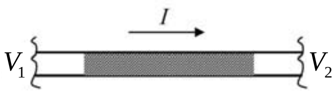
$$\varepsilon_n = n^2\frac{\hbar^2\pi^2}{2mW^2}$$



$$G = \frac{2e^2}{h}\sum_n f(\varepsilon_n)$$

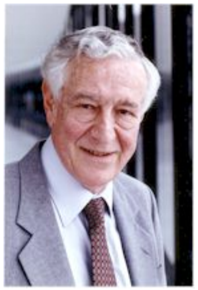
Characteristic energy : $\varepsilon^* = \frac{\hbar^2\pi^2}{2m^*W^2} \sim 1K$ $\quad W \sim 250nm$

Landauer-Büttiker multiterminal formalism

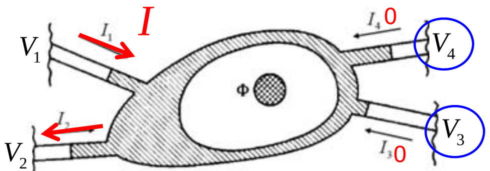


$$G = 2\frac{e^2}{h}T$$

Landauer formula



R. Landauer (1927-1999)



Current probes

Voltage probes

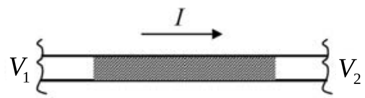
$$G_{12,34} = \frac{I_{12}}{V_3 - V_4}$$

M. Büttiker (1950-2013)

M. Büttiker, Four terminal Phase-coherent conductor, PRL 1986

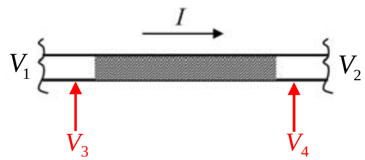
Landauer-Büttiker formulae

Two-terminal conductance

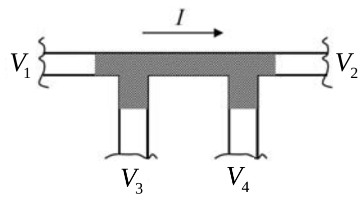


$$G_2 = 2\frac{e^2}{h}T$$

Four-terminal conductance



$$G_4 = 2\frac{e^2}{h}\frac{T}{R}$$

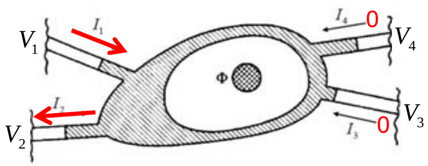


$$G_4 = 2\frac{e^2}{h}?$$

How many coefficients to characterize the « black box » ?

$$I_i = \frac{2e^2}{h}\left[(M_i - R_{ii})V_i - \sum_{j\neq i} T_{ij}V_j\right]$$

M. Büttiker



$$I = \bar{G}V$$

Conductance matrix

$$\bar{G} = \frac{2e^2}{h}\begin{pmatrix} M_1 - R_{11} & T_{12} & T_{13} & T_{14} \\ -T_{21} & M_2 - R_{22} & -T_{23} & -T_{24} \\ -T_{31} & -T_{32} & M_3 - R_{33} & -T_{34} \\ -T_{41} & -T_{42} & -T_{43} & M_4 - R_{44} \end{pmatrix}$$

Four terminals

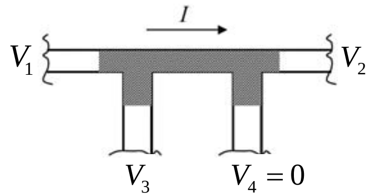
9 transmission coefficients…

$$\begin{pmatrix} I \\ -I \\ 0 \\ 0 \end{pmatrix} = \frac{2e^2}{h}\begin{pmatrix} M_1 - R_{11} & T_{12} & T_{13} & T_{14} \\ -T_{21} & M_2 - R_{22} & -T_{23} & -T_{24} \\ -T_{31} & -T_{32} & M_3 - R_{33} & -T_{34} \\ -T_{41} & -T_{42} & -T_{43} & M_4 - R_{44} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

$$V_4 = 0$$

$$\begin{pmatrix} -I \\ 0 \\ 0 \end{pmatrix} = \frac{2e^2}{h}\begin{pmatrix} -T_{21} & M_2 - R_{22} & -T_{23} \\ -T_{31} & -T_{32} & M_3 - R_{33} \\ -T_{41} & -T_{42} & -T_{43} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

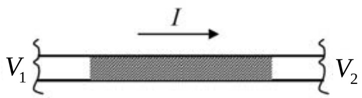
$$V_3 = \frac{h}{2e^2}\frac{T_{31}T_{42} - T_{32}T_{41}}{D}I$$



$$G_{12,34} = \frac{I_{12}}{V_3 - V_4}$$

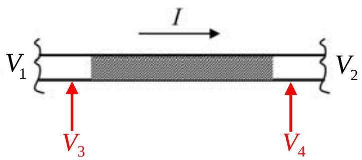
Landauer-Büttiker formulae

Two-terminal conductance

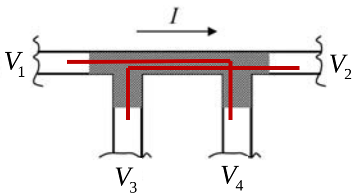


$$G_2 = 2\frac{e^2}{h}T$$

Four-terminal conductance



$$G_4 = 2\frac{e^2}{h}\frac{T}{R}$$



$$G_4 = 2\frac{e^2}{h}\frac{D}{T_{31}T_{42} - T_{32}T_{41}}$$

can be negative !

In general depends on 9 transmission coefficients…

Time Reversal Symmetry

M. Büttiker

$$\bar{G} = \frac{2e^2}{h}\begin{pmatrix} M_1 - R_{11} & T_{12} & T_{13} & T_{14} \\ -T_{21} & M_2 - R_{22} & -T_{23} & -T_{24} \\ -T_{31} & -T_{32} & M_3 - R_{33} & -T_{34} \\ -T_{41} & -T_{42} & -T_{43} & M_4 - R_{44} \end{pmatrix}$$

$$T_{ij}(B) = T_{ji}(-B)$$

$$B = 0 \qquad T_{ij} = T_{ji}$$

$$\bar{G} = \frac{2e^2}{h}\begin{pmatrix} M_1 - R_{11} & -T_{12} & -T_{13} & -T_{14} \\ -T_{21} & M_2 - R_{22} & -T_{23} & -T_{24} \\ -T_{31} & -T_{32} & M_3 - R_{33} & -T_{34} \\ -T_{41} & -T_{42} & -T_{43} & M_4 - R_{44} \end{pmatrix}$$

4 terminals | **N terminals**

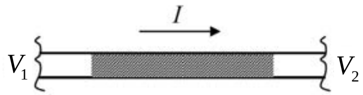
9 transmission coefficients… $(N-1)^2$

In zero field, the 3 x 3 submatrix is symmetric

6 transmission coefficients… $\frac{N(N-1)}{2}$

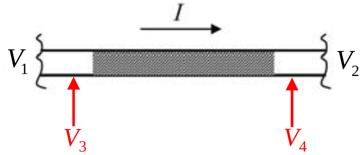
Landauer-Büttiker formulae

Two-terminal conductance

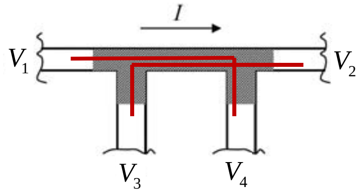


$$G_2 = 2\frac{e^2}{h}T$$

Four-terminal conductance



$$G_4 = 2\frac{e^2}{h}\frac{T}{R}$$

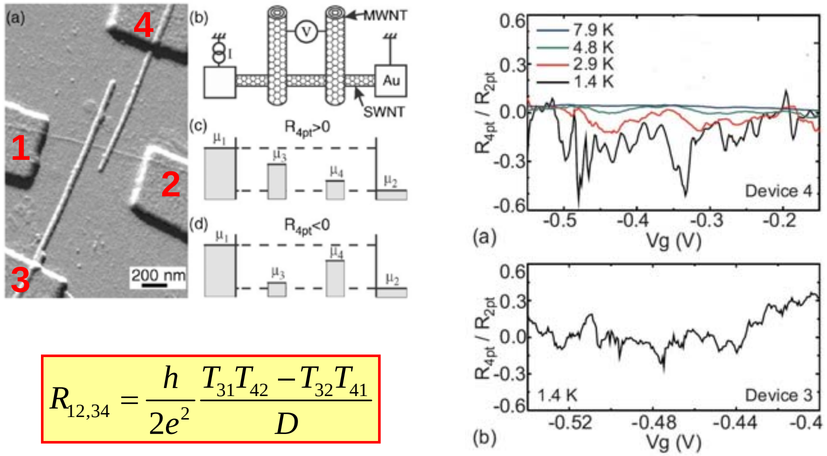


$$G_4 = 2\frac{e^2}{h}\frac{D}{T_{31}T_{42} - T_{32}T_{41}}$$

In general depends on 9 transmission coefficients… (6 in zero field)

$T_{ij}(B) = T_{ji}(-B)$ $\quad$ $T_{ij} = T_{ji}$

4 terminal resistance in a carbon nanotube

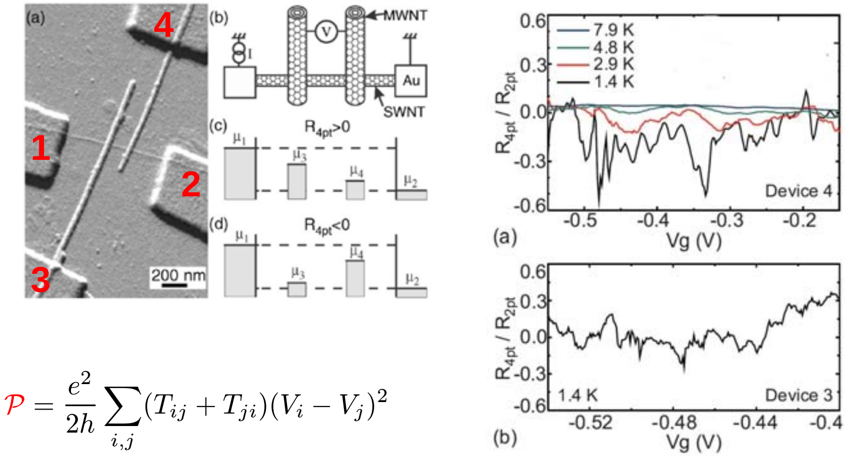


$$R_{12,34} = \frac{h}{2e^2}\frac{T_{31}T_{42} - T_{32}T_{41}}{D}$$

Low T : the 4 terminal resistance can be negative

B. Gao *et al.*, *Four-point resistance of individual single-wall carbon nanotubes*, Phys. Rev. Lett. 95, 196802 (2005)

4 terminal resistance in a carbon nanotube

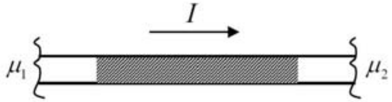


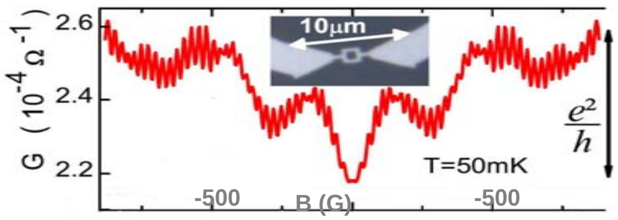
$$\mathcal{P} = \frac{e^2}{2h}\sum_{i,j}(T_{ij} + T_{ji})(V_i - V_j)^2$$

Low T : the 4 terminal resistance can be negative,
but power dissipate is positive

Symmetry of the two-terminal conductance

$$T(B) = T(-B)$$

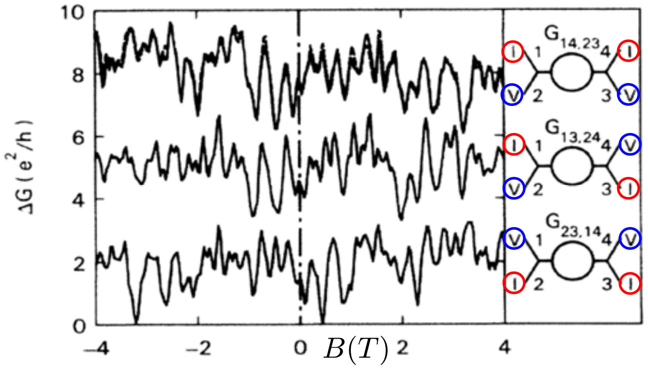




$$G(B) = G(-B)$$

L. Angers et al., Phys. Rev. B 75, 115309 (2007)

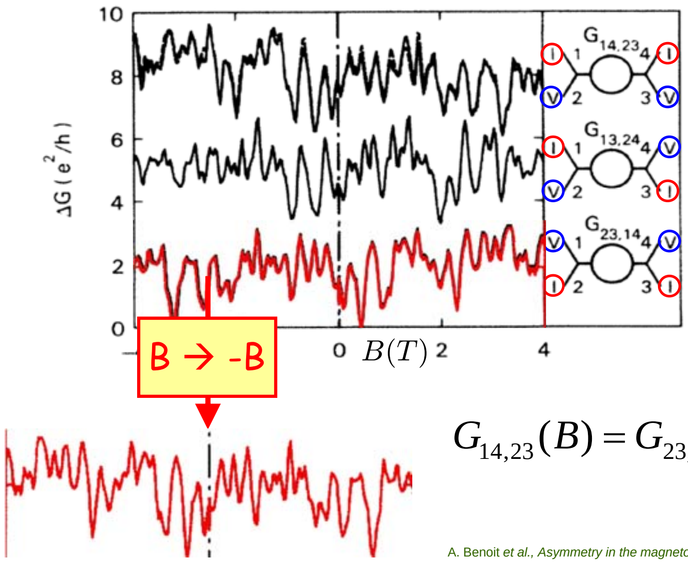
Symmetry of the four-terminal conductance ?



$$G_{12,34} = \frac{I_{12}}{V_{34}} = 2\frac{e^2}{h}\frac{D}{T_{31}T_{42} - T_{32}T_{41}}$$

A. Benoit *et al., Asymmetry in the magnetoconductance of metal wires and loops*, Phys. Rev. Lett. 57, 1765 (1986)

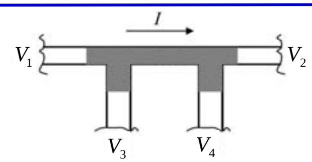
Symmetry of the four-terminal conductance ?



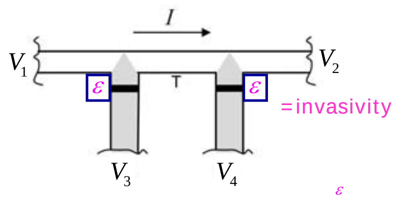
$$G_{14,23}(B) = G_{23,14}(-B)$$

A. Benoit *et al., Asymmetry in the magnetoconductance of metal wires and loops*, Phys. Rev. Lett. 57, 1765 (1986)

Landauer-Büttiker formulae

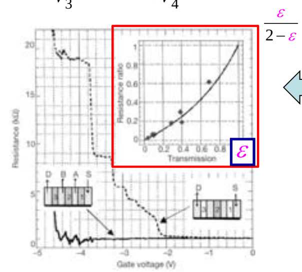


In de Picciotto experiment, 6 coefficients reduce to one



$$G_4 = 2\frac{e^2}{h}\frac{D}{T_{31}T_{42} - T_{32}T_{41}}$$

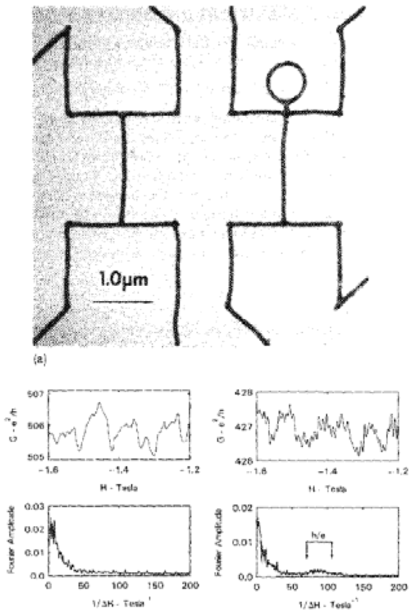
$$T_{31} = T_{42} = \varepsilon$$
$$T_{32} = T_{41} = \varepsilon(1-\varepsilon)$$
$$T_{34} = \varepsilon^2$$
$$T_{12} = (1-\varepsilon)^2$$



$$G_4(\varepsilon) = 2\frac{e^2}{h}\frac{2-\varepsilon}{\varepsilon}$$

Phase coherence

Non- locality

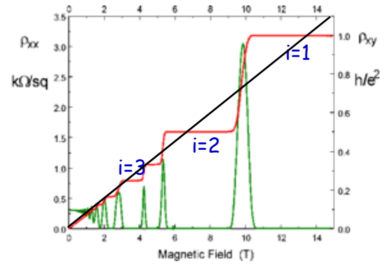


Nonlocal electrical properties in mesoscopic devices

C. P. Umbach, P. Santhanam, C. van Haesendonck, and R. A. Webb
IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

Appl. Phys. Lett. 50, 1289 (1987)

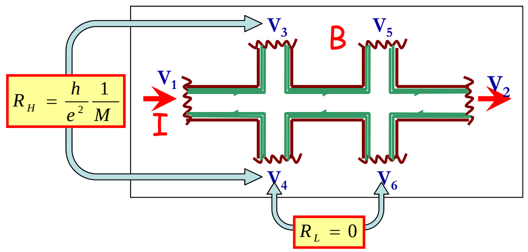
Quantum Hall Effect



R_K=25 812, 807 Ω

$$R_H = \frac{h}{e^2}\frac{1}{M}$$

$$R_L = 0$$

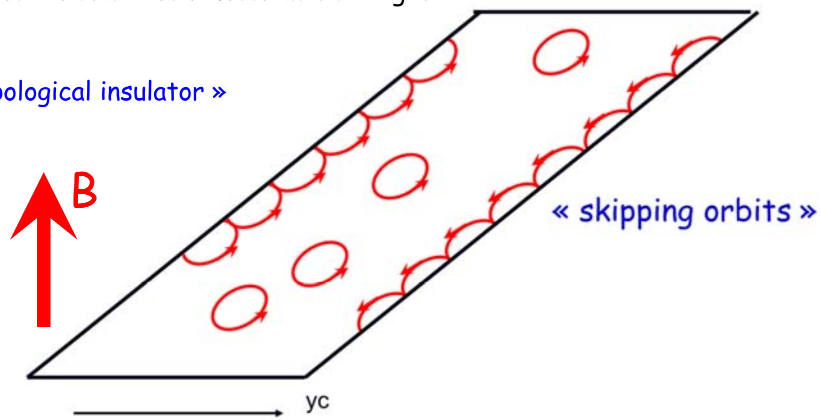


Quantum Hall effect

Bulk trajectories are pinned by disorder
Chiral edge trajectories propagate freely

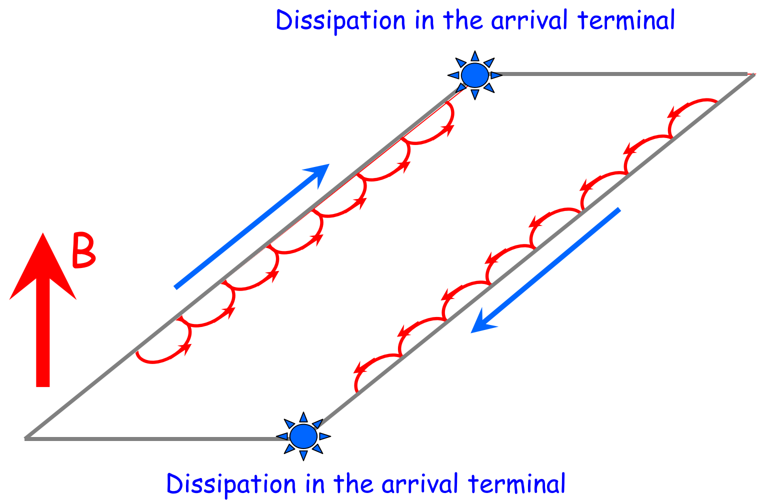
Bulk insulator
Perfect « chiral » conductor at the edges

« Topological insulator »



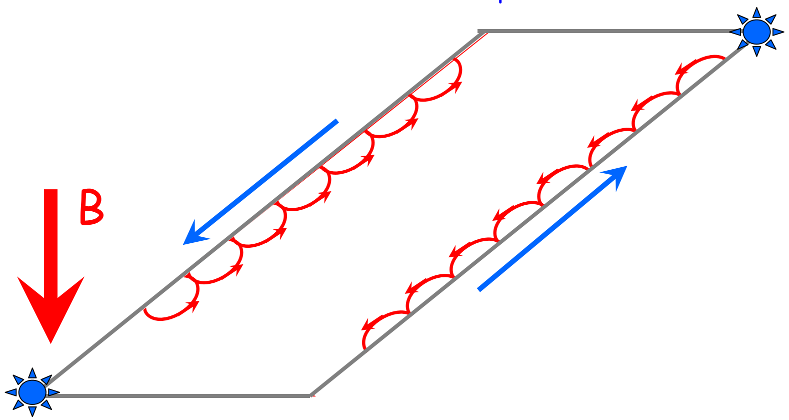
Quantum Hall effect

Left-going and right-going electrons are <u>spatially separated</u>



Quantum Hall effect

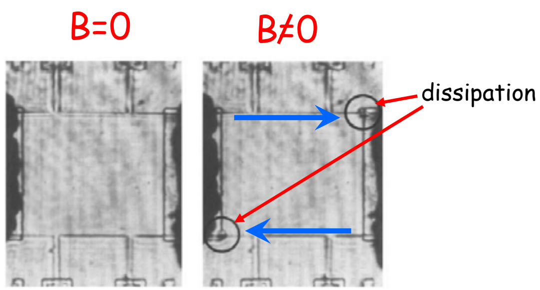
Left-going and right-going electrons are <u>spatially separated</u>



Quantum Hall effect

Left-going and right-going electrons are <u>spatially separated</u>

This experiment shows that electrons stay at the chemical potential of the injection reservoir and exchange their energy at the arrival reservoir



Imaging of the dissipation in quantum Hall effect experiments
U. Klass et al., Z. Phys. B 82, 351 (1991)