Lecture 4

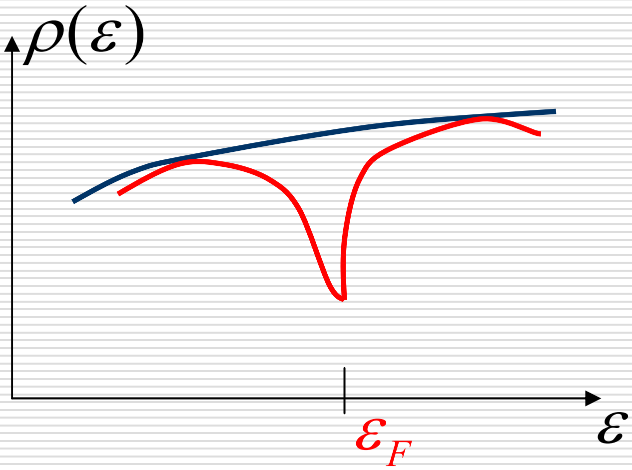
Diffusion + e-e interations

Density of states anomaly
Dephasing due to electron-electron interactions

e-e interaction

Landau Fermi liquid picture $$\frac{1}{\tau_{ee}(\varepsilon)} \propto \varepsilon^2$$

Diffusion slows down electrons $$\frac{1}{\tau_{ee}(\varepsilon)} \propto \varepsilon^{d/2}$$



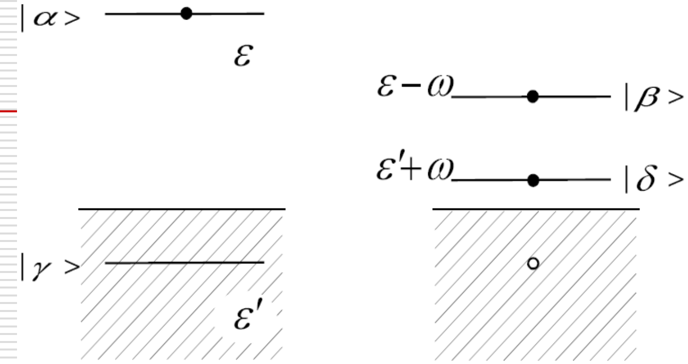
Density of states anomaly near ε_F

Correction to conductivity

Dephasing by e-e interactions

$$L_\phi(T) \quad ??$$

Lifetime of quasiparticle



Landau | Fermi golden rule

$$\frac{1}{\tau_{ee}(\varepsilon)} \propto \int_0^{\varepsilon} W^2 \omega \, d\omega \sim \varepsilon^2$$

$$-\omega < \varepsilon' < 0$$

$$\varepsilon < \omega$$

W : matrix element of the interaction

Diffusive conductors : the typical matrix element of the interaction is energy dependent **(Altshuler-Aronov)**

The effective e-e interaction is proportional to the time spent in the interaction region :

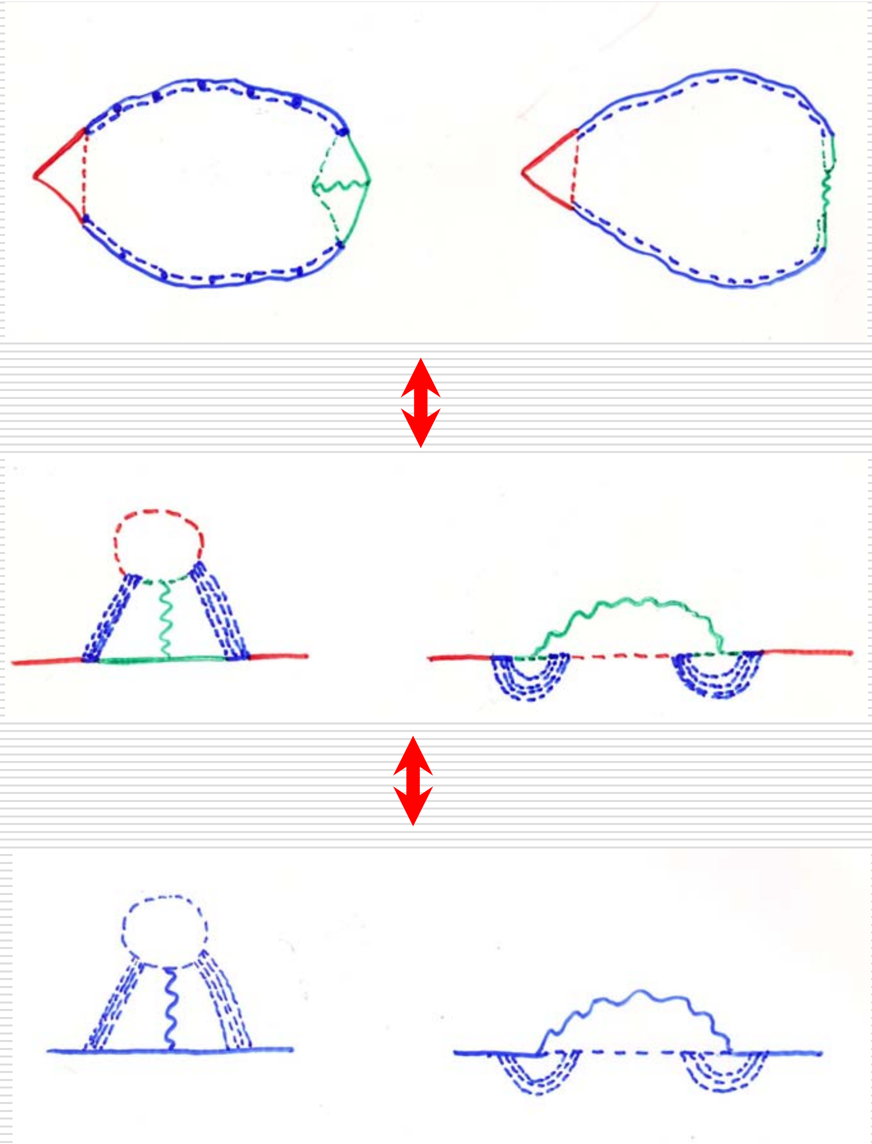
$$W^2(\omega) \propto \int^{1/\omega} t \, P(t) \, dt \qquad P(t) = \frac{1}{(4\pi D t)^{d/2}}$$

$P(t)$ return probability

$$W^2(\omega) \propto \frac{1}{\omega^{2-d/2}}$$

$$\frac{1}{\tau_{ee}(\varepsilon)} \propto \varepsilon^{d/2}$$

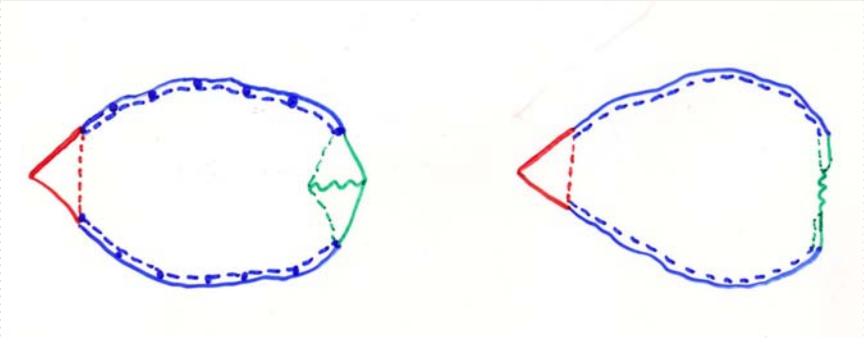
Density of states anomaly



Hartree

Fock (exchange)

Density of states anomaly



$\delta\rho(\varepsilon)$ involves two electrons with energy difference ε

They stay in phase during a time $\hbar/\varepsilon$

$$\frac{\delta\rho(\varepsilon)}{\rho} \propto \text{ probability to have loops of time } t < \hbar/\varepsilon$$

$$\frac{\delta\rho(\varepsilon)}{\rho} \propto \lambda_\rho \frac{\lambda_F^{d-1} v_F}{V} \int_{\tau_e}^{\hbar/\varepsilon} P(t)dt$$

Density of states anomaly

$$\frac{\delta\rho(\varepsilon)}{\rho} \propto \lambda_\rho \frac{\lambda_F^{d-1} v_F}{V} \int_{\tau_e}^{\hbar/\varepsilon} P(t)dt = \frac{\lambda_\rho}{g} \int_{\tau_e}^{\hbar/\varepsilon} P(t)\frac{dt}{\tau_D}$$

$$\tau_\phi \to \hbar/\varepsilon$$

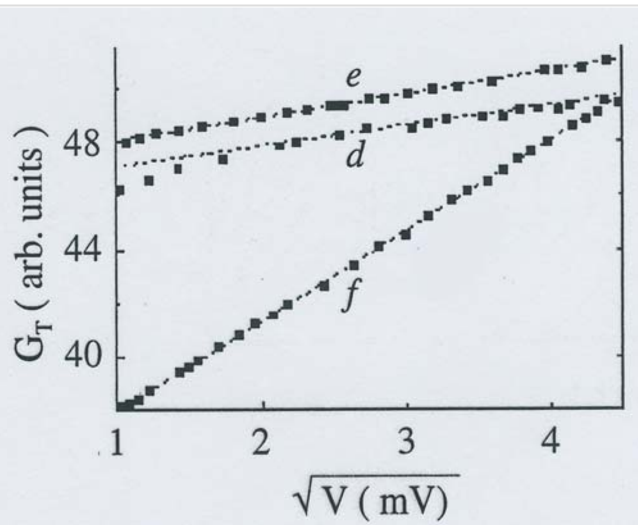
$$L_\phi \to L_\varepsilon = \sqrt{\frac{\hbar D}{\varepsilon}}$$

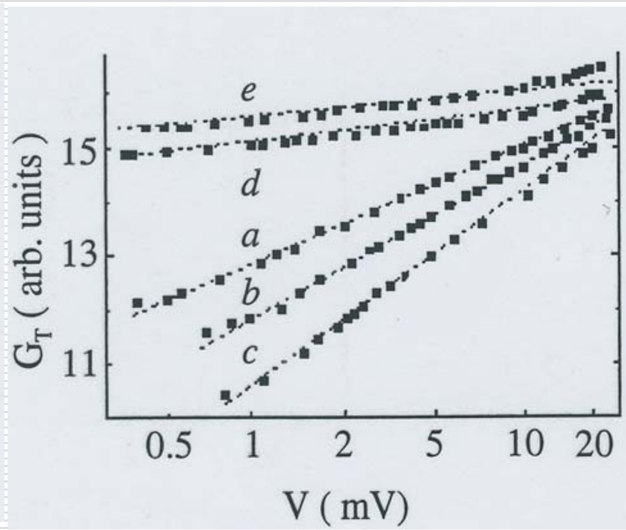
$$\delta\rho(\varepsilon) \propto -\frac{1}{D} \times \begin{cases} L_\varepsilon - l_e & d=1 & -\frac{1}{\sqrt{D\varepsilon}} \\ \ln\frac{L_\varepsilon}{l_e} & d=2 & \frac{1}{D}\ln\varepsilon\tau_e \\ \frac{1}{l_e} - \frac{1}{L_\varepsilon} & d=3 & -C + \frac{\sqrt{\varepsilon}}{D^{3/2}} \end{cases}$$

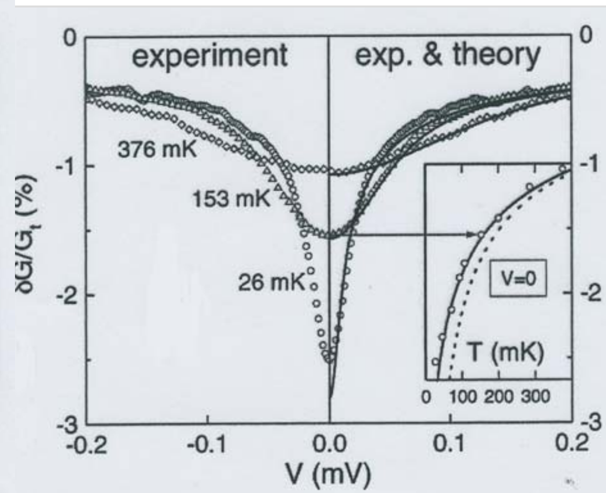
Density of states anomaly → Tunnel conductance anomaly

Tunnel conductance

$$\frac{\delta G_t(V)}{G_t} \propto \frac{\delta\rho(\varepsilon = eV)}{\rho}$$







Thick film (3d)

Thin film (2d)

Wire (1d)

Imry, Ovadyahu

Saclay group

Correction to the conductivity

$$\sigma(\varepsilon) = e^2 D \rho(\varepsilon)$$

$$\delta\sigma(T) = \int \left(-\frac{\partial f}{\partial \varepsilon} \right) \sigma(\varepsilon) d\varepsilon$$

Anomaly in $\rho(\varepsilon)$ → correction $\delta\sigma(T)$

$$\varepsilon \to k_B T$$

$$L_\varepsilon \sqrt{\frac{\hbar D}{\varepsilon}} \to L_T = \sqrt{\frac{\hbar D}{k_B T}}$$

$$\delta\sigma(T) \propto -\frac{1}{D} \times \begin{cases} L_T - l_e & d=1 & -\frac{1}{\sqrt{DT}} \\ \ln \frac{L_T}{l_e} & d=2 & \frac{1}{D} \ln T\tau_e \\ \frac{1}{l_e} - \frac{1}{L_T} & d=3 & -C + \frac{\sqrt{T}}{D^{3/2}} \end{cases}$$

Summary of quantum corrections

$$\delta X \propto \begin{cases} L_X - l_e & d=1 \\ \ln \dfrac{L_X}{l_e} & d=2 \\ \dfrac{1}{l_e} - \dfrac{1}{L_X} & d=3 \end{cases}$$

Corrections of order 1/g

$$L_X = L, \quad L_\phi = \sqrt{D\tau_\phi}, \quad L_B = \sqrt{\frac{\hbar}{eB}}, \quad L_\omega = \sqrt{\frac{D}{\omega}}$$

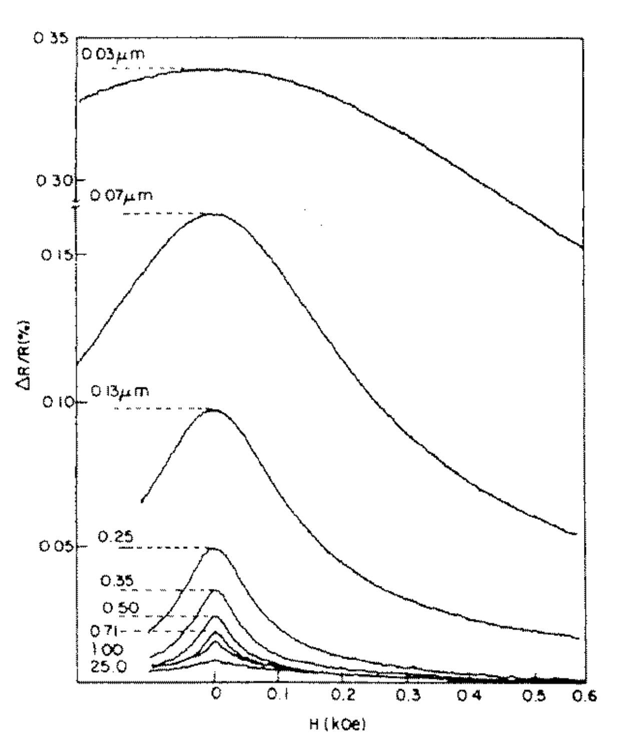
$$L_\varepsilon = \sqrt{\frac{\hbar D}{\varepsilon}} = \sqrt{\frac{\hbar D}{eV}} \qquad L_T = \sqrt{\frac{\hbar D}{k_B T}}$$

Dephasing by e-e interactions

Temperature dependence of the phase coherence length

$L_\phi(T)$??

W.L. in a quasi-1D wire



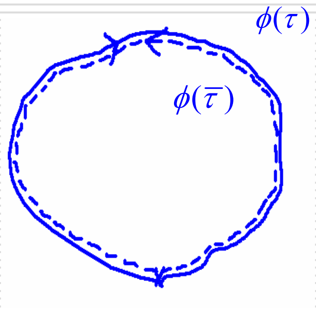
Licini,Dolan,Bishop,1980

$$B^* W L_\phi(T) \sim \phi_0 \rightarrow L_\phi \propto T^{-1/3}$$

Dephasing by e-e interactions

Weak-localization correction :

$$\Delta g = -2s\int P_0(t)\ e^{-t/\tau_\phi}\ \frac{dt}{\tau_D}$$



$$\Delta g = -2s\int P_0(t)\ \left\langle e^{i\Phi(t)} \right\rangle \frac{dt}{\tau_D}$$

Dephasing : e-e interaction

quasi-1D wire

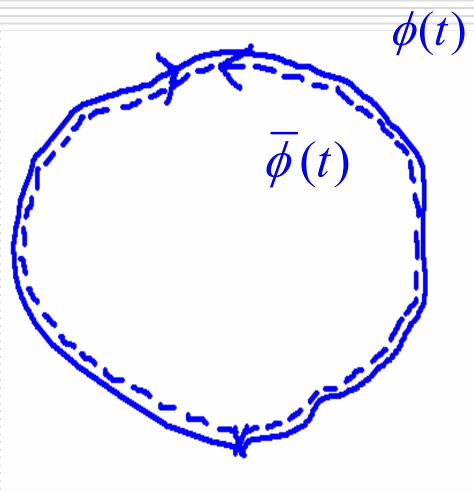
Phase coherence time $\tau_\phi \propto T^{-2/3}$

Altshuler,Aronov,Khmelnitskii

$\tau_\phi(T) \propto T^{-2/3}$: a qualitative derivation

quasi-1d wire

$$\left\langle e^{i\Phi(t)} \right\rangle \quad ?$$



e-e interaction = electric fluctuating potential → Fluctuating phase

$$\left\langle e^{i\Phi(t)} \right\rangle \sim e^{-\frac{1}{2}\left\langle \Phi^2(t) \right\rangle}$$

$$\Phi(t) = \phi(t) - \bar{\phi}(t)$$

$$\phi(t) = \frac{e}{\hbar}\int_0^t V(r(\tau),\tau)\, d\tau$$

$$\left\langle \Phi^2(t) \right\rangle = 2\frac{e^2}{\hbar^2}\int_0^t \left[\left\langle V(r_\tau,\tau)V(r_\tau,\tau) \right\rangle - \left\langle V(r_\tau,\tau)V(r_\tau,\bar{\tau}) \right\rangle \right] d\tau$$

$$\frac{d\left\langle \Phi^2(t) \right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\left\langle V^2 \right\rangle_t$$

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$$\left\langle V^2\right\rangle_t \sim k_B T\, R_t \sim k_B T\,\frac{r_t}{\sigma_0 S}$$

$$\frac{d\left\langle \Phi^2(t)\right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0 S}\; r_t$$

Diffusion

$$r_t \sim \sqrt{Dt}$$

$$\left\langle \Phi^2(t)\right\rangle \sim \frac{k_B T\sqrt{D}}{\sigma_0 S}\; t^{3/2} \sim \left(\frac{t}{\tau_N}\right)^{3/2}$$

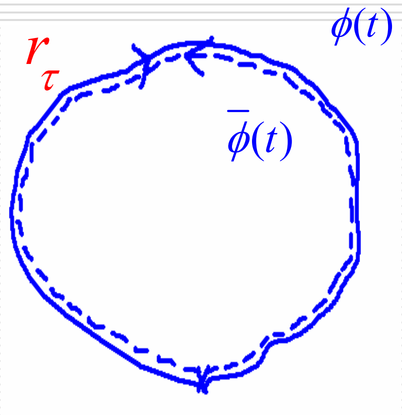
$$\tau_N \sim \left(\frac{\sigma_0 S}{e^2 k_B T\sqrt{D}}\right)^{2/3} \propto \frac{1}{T^{2/3}}$$

Nyquist time (Aronov, Altshuler, Khmelnitskii)

$$\left\langle e^{i\Phi(t)}\right\rangle \sim \left\langle e^{-\frac{1}{2}\Phi^2(t)}\right\rangle \sim e^{-(t/\tau_N)^{3/2}}$$

$\tau_\phi(T) \propto T^{-2/3}$: a detailed derivation

quasi-1d wire



$$\left\langle e^{i\Phi(t)} \right\rangle_T \qquad \Phi(t) = \phi(t) - \bar{\phi}(t)$$

1) Phase fluctuations originate from potential fluctuations

$$\phi(t) = \frac{e}{\hbar}\int_0^t V(r_\tau,\tau)\, d\tau$$

2) Characterize potential fluctuations ***Nyquist***

$$\left\langle [V(0)-V(L)]^2 \right\rangle_T (\omega) = 2\, k_B T\, R = \frac{2\, k_B T\, L}{\sigma S}$$

$$\left\langle [V(r)-V(r')]^2 \right\rangle_T (\omega) = \frac{2\, k_B T\, |r-r'|}{\sigma S}$$

$$\left\langle [V(r,\tau)-V(r',\tau)][V(r,\tau')-V(r',\tau')] \right\rangle_T = \frac{2 k_B T\, |r-r'|}{\sigma S}\delta(\tau-\tau')$$

$\tau_\phi(T) \propto T^{-2/3}$: a detailed derivation

quasi-1d wire

$$\frac{1}{2}\left\langle \Phi^2(t) \right\rangle_T = \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma S}\int \left| r(\tau) - r(\bar{\tau}) \right| d\tau = \frac{1}{\sqrt{D\tau_N^{3/2}}}\int_0^t \left| r(\tau) - r(\bar{\tau}) \right| d\tau \propto \left(\frac{t}{\tau_N}\right)^{3/2}$$

3) Characteristic time

$$\tau_N^{3/2} = \frac{\hbar^2 \sigma S}{e^2 \sqrt{D} k_B T}$$

Nyquist time

4) Gaussian fluctuations of electromagnetic potential

$$\left\langle \left\langle e^{i\Phi(t)} \right\rangle_T \right\rangle_C = \left\langle e^{-\frac{1}{2}\left\langle \Phi^2(t) \right\rangle_T} \right\rangle_C$$

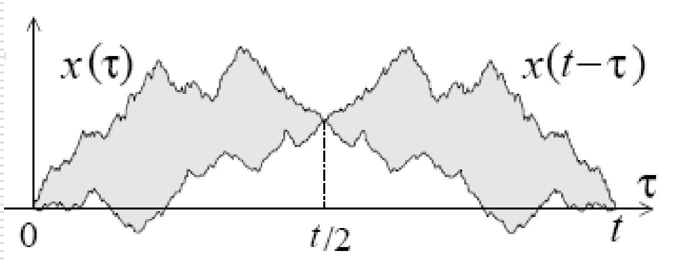
Thermal fluctuations

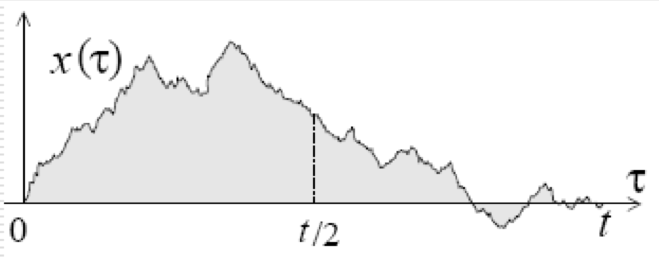
trajectories

$$< e^{-\frac{1}{\sqrt{D\tau_N^{3/2}}}\int_0^t \left| r(\tau) - r(\bar{\tau}) \right| d\tau} >_C$$

5) Property of a Brownian bridge

$$< e^{-\frac{1}{\sqrt{D}\tau_N^{3/2}}\int_0^t |r(\tau)-r(\overline{\tau})|d\tau} >_C \;=\; < e^{-\frac{1}{\sqrt{D}\tau_N^{3/2}}\int_0^t |r(\tau)|d\tau} >_C$$





$$\int_0^t d\tau\, |x(\tau) - x(t-\tau)| \overset{(\text{law})}{=} \int_0^t d\tau\, |x(\tau)|$$

Comtet et al.

$$\left\langle e^{i\Phi(t)} \right\rangle = < e^{-\frac{1}{\sqrt{D}\tau_N^{3/2}}\int_0^t |r(\tau)|d\tau} >_C$$

6) Path integral formulation

$$\left(\frac{\partial}{\partial t} - D\Delta + U(r)\right) P(r,r',t) = \delta(r-r')\delta(t)$$

$$P(r,r',t) = \int_{r(0)=r}^{r(t)=r'} D\{r\}\, e^{-\int_0^t [(\frac{\dot{r}^2}{4D}) + U(r)] d\tau}$$

6) Path integral formulation

$$< e^{-\frac{1}{\sqrt{D}\tau_N^{3/2}}\int_0^t |r(\tau)| d\tau} >_C = \frac{\int_{r(0)=r}^{r(t)=r} D\{r\}\, e^{-\int_0^t [(\frac{\dot{r}^2}{4D\tau}) + \frac{1}{\sqrt{D}\tau_N^{3/2}}|r(\tau)|] d\tau}}{P_0(r,r,t)}$$

$$P(r,r,t) = P_0(r,r,t)\left\langle e^{i\Phi(t)} \right\rangle$$ is solution of :

$$\left(\frac{\partial}{\partial t} - D\Delta + \frac{1}{\sqrt{D}\tau_N^{3/2}}|r| \right) P(r,r',t) = \delta(r-r')\delta(t)$$

7) Laplace transform

$$P(r,r,\gamma) = \int P_0(r,r,t)\left\langle e^{i\Phi(t)}\right\rangle e^{-\gamma t}dt$$ is solution of :

$$\left(\gamma - D\Delta + \frac{1}{\sqrt{D}\tau_N^{3/2}}|r|\right)P(r,r',\gamma) = \delta(r-r')$$

8) Dimensionless differential equation $r = x\sqrt{D\tau_N}$

$$\left(\gamma\tau_N - \frac{\partial^2}{\partial x^2} + |x|\right)P(x,x',\gamma) = \sqrt{\frac{\tau_N}{D}}\;\delta(x-x')$$

9) Solve differential equation

$$\left(\gamma\tau_N - \frac{\partial^2}{\partial x^2} + |x| \right) P(x,x',\gamma) = \sqrt{\frac{\tau_N}{D}}\ \delta(x-x')$$

$$P(x,x,\gamma) = -\frac{1}{2}\sqrt{\frac{\tau_N}{D}}\ \frac{\mathrm{Ai}(\gamma\tau_N)}{\mathrm{Ai}'(\gamma\tau_N)}$$

$$\Delta g = -2s\int P_0(t)\ \left\langle e^{i\Phi(t)} \right\rangle e^{-\gamma t}\frac{dt}{\tau_D} = -2s\frac{D}{L}P(x,x,\gamma)$$

$$\Delta g = s\frac{\sqrt{D\tau_N}}{L}\ \frac{\mathrm{Ai}(\gamma\tau_N)}{\mathrm{Ai}'(\gamma\tau_N)}$$

$$\Delta g = s \frac{\sqrt{D\tau_N}}{L} \frac{\mathrm{Ai}(\gamma\tau_N)}{\mathrm{Ai}'(\gamma\tau_N)}$$

$$\frac{\mathrm{Ai}(x)}{\mathrm{Ai}'(x)} \approx -\frac{1}{\sqrt{1/2+x}}$$

Assuming exponential relaxation, we had obtained

$$\Delta g = -2s\int P_0(t)\, e^{-t/\tau_\phi - \gamma t} \frac{dt}{\tau_D}$$

$$\Delta g = -s\frac{\sqrt{D}}{L}\left(\frac{1}{\tau_\phi}+\gamma\right)^{-1/2}$$

$$\Delta g \approx -s\frac{\sqrt{D}}{L}\left(\frac{1}{2\tau_N}+\gamma\right)^{-1/2}$$

Conclusion:

exponential relaxation with $\tau_\phi = 2\tau_N$ is a very good approximation

Dephasing by e-e interactions is very well described by an exponential relaxation

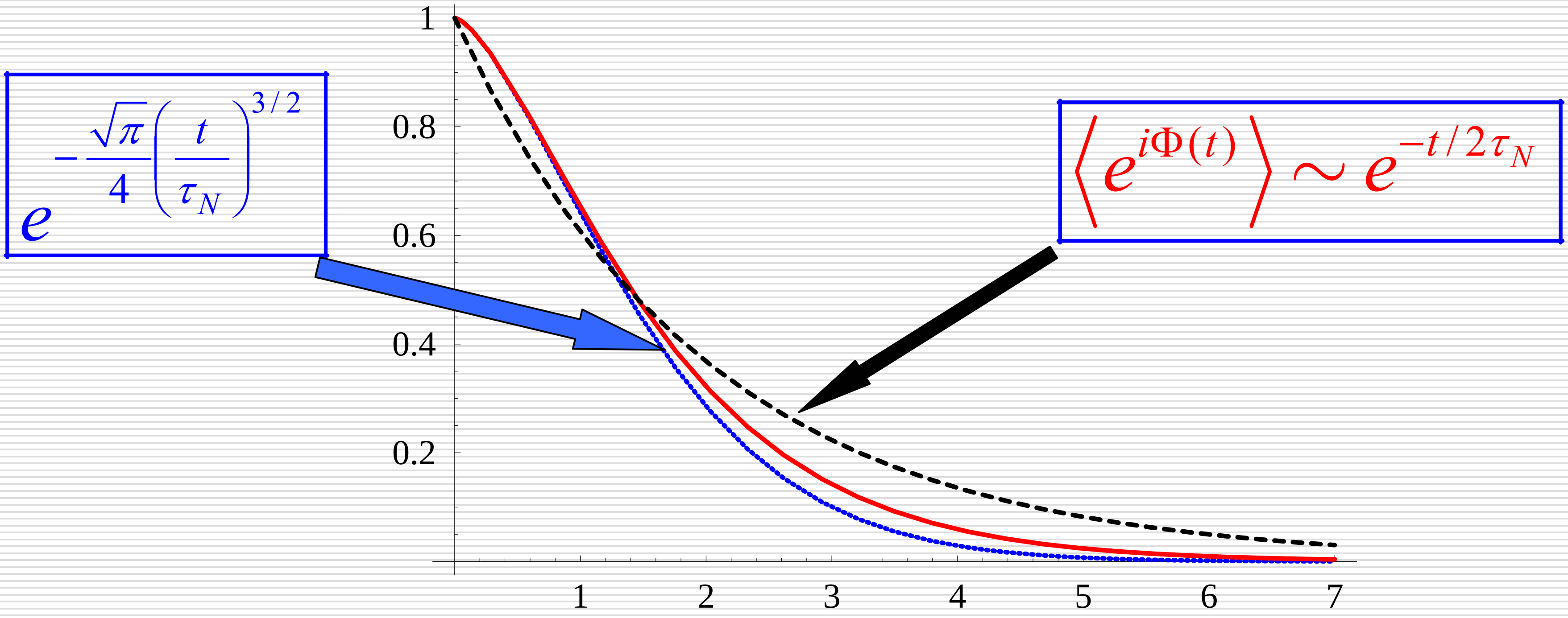
$$\Delta g = -2s \int P_0(t)\ e^{-t/\tau_\phi - \gamma t}\ \frac{dt}{\tau_D}$$

$$\tau_\phi = 2\tau_N = 2\left(\frac{\hbar^2 \sigma S}{e^2 \sqrt{D k_B T}}\right)^{2/3}$$

$$k_B T \frac{\tau_N}{\hbar} = \frac{\hbar\ \sigma S}{e^2 \sqrt{D \tau_N}} = \frac{\hbar \sigma S}{e^2 L_N} = g(L_N)$$

$$\frac{\hbar}{\tau_N} = \frac{k_B T}{g(L_N)}$$

$$\left\langle e^{i\Phi(t)} \right\rangle = f\left(\frac{t}{\tau_N}\right)$$



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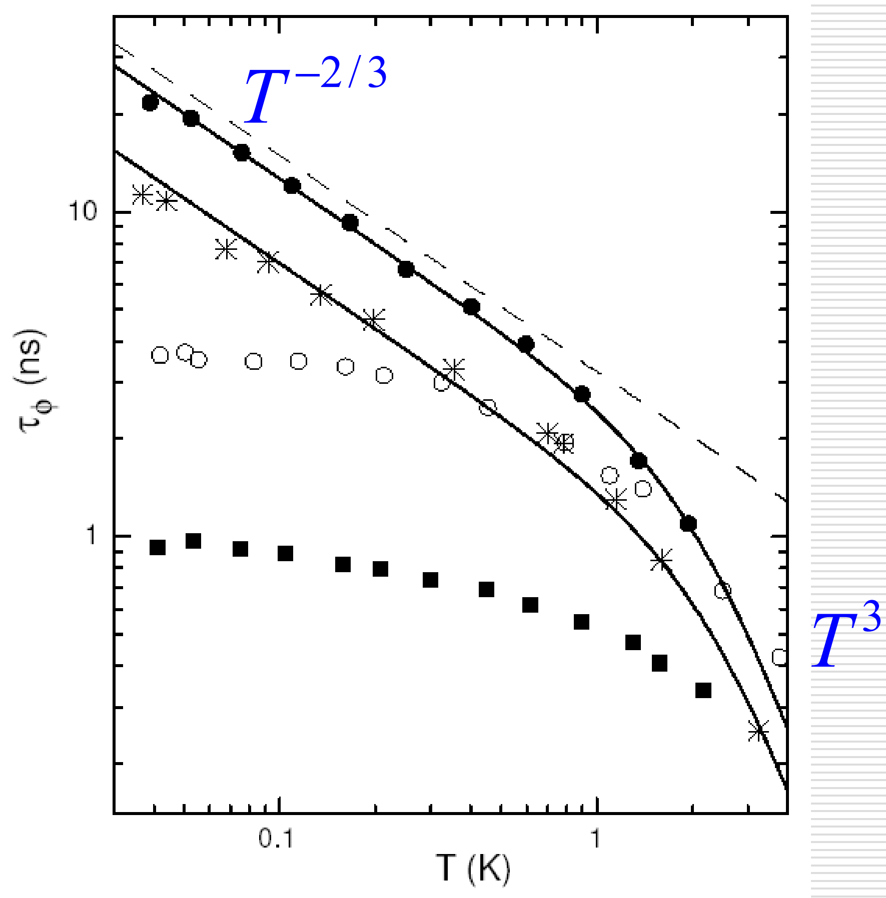

Saclay group

$$\frac{1}{\tau_\phi(T)} = AT^{2/3} + BT^3$$