e-e interaction correction
to the conductivity of metallic networks

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Disorder + interaction

Each electron is scattered by disordered potential
+ electrostatic potential created by other electrons

↓

interaction induced correction on classical conductivity
depends on spatial dimensionality

Altshuler,Aronov,Lee '80; Altshuler, Aronov '85

Finkelshtein '83

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Disorder + interaction

The interaction induced correction on classical conductivity depends on spatial dimensionality : *Altshuler,Aronov,Lee '80; Altshuler, Aronov '85*

wire :

$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{h} 0.782 L_T$$

$$L_T = \sqrt{\frac{\hbar D}{k_B T}}$$

$$L_T^{2-d}$$

2D :

$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{\pi h} \ln\frac{L_T}{l_e}$$

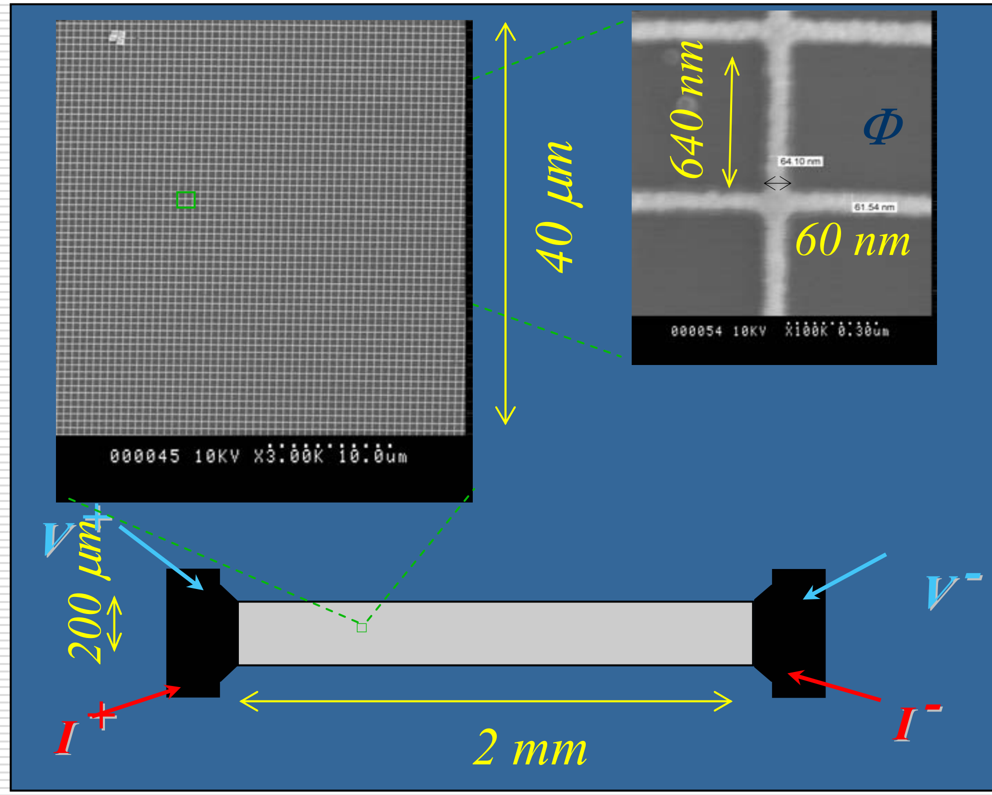
$$P(t) = \frac{1}{(4\pi D t)^{d/2}}$$

Network of quasi-1D wires ?

Correction on networks?

Bauerle, Mallet, Mailly, Saminadayar, Schopfer, GRENOBLE

Angers, Bouchiat, Ferrier, Guéron, ORSAY



$$\Delta\sigma_{ee}(L_T)??$$

Network of quasi-1D wires ?

Goal:

Write the correction $\Delta\sigma_{ee}$ to the conductivity in a way such its calculation can be easily generalized to any geometry.

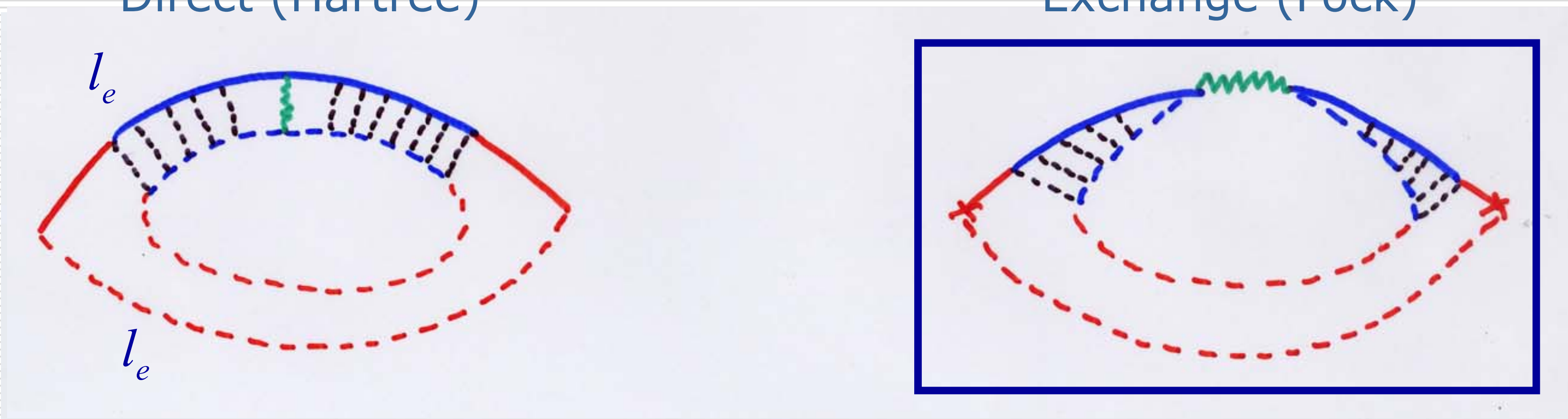
Cf. Weak localization :

$$\Delta\sigma_{WL} = -\frac{2e^2 D}{\pi}\int_0^\infty P_c(t)\, e^{-t/\tau_\phi}\, dt$$

$P_c(t)$ "cooperon" return probability

Reminder: physical origin

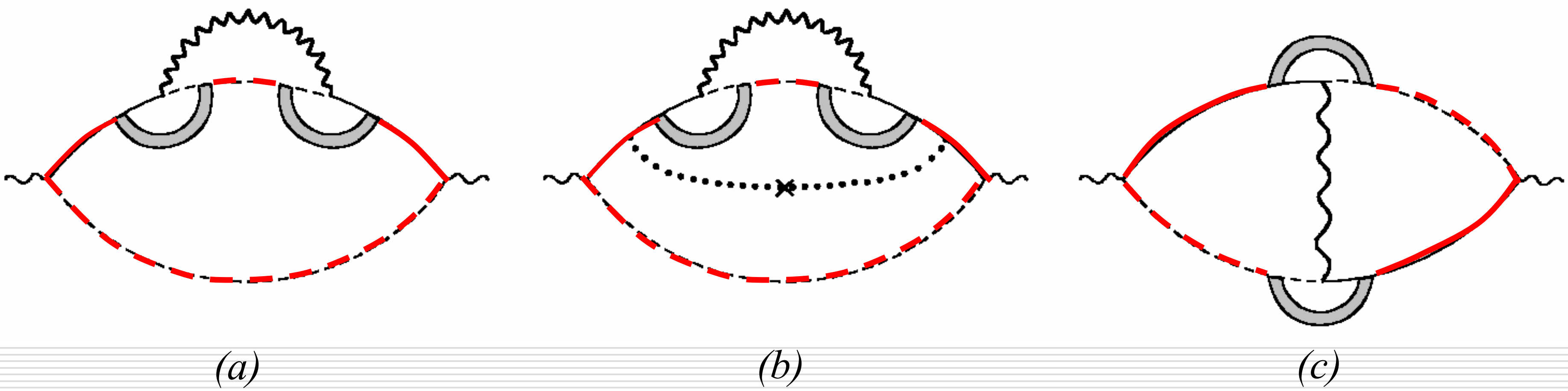
Direct (Hartree) | Exchange (Fock)

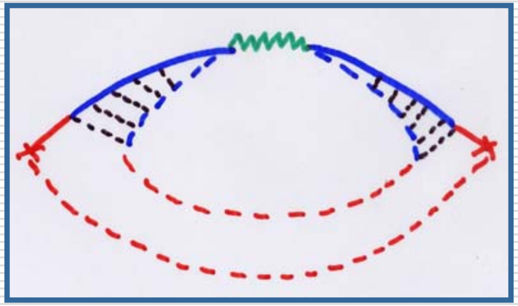


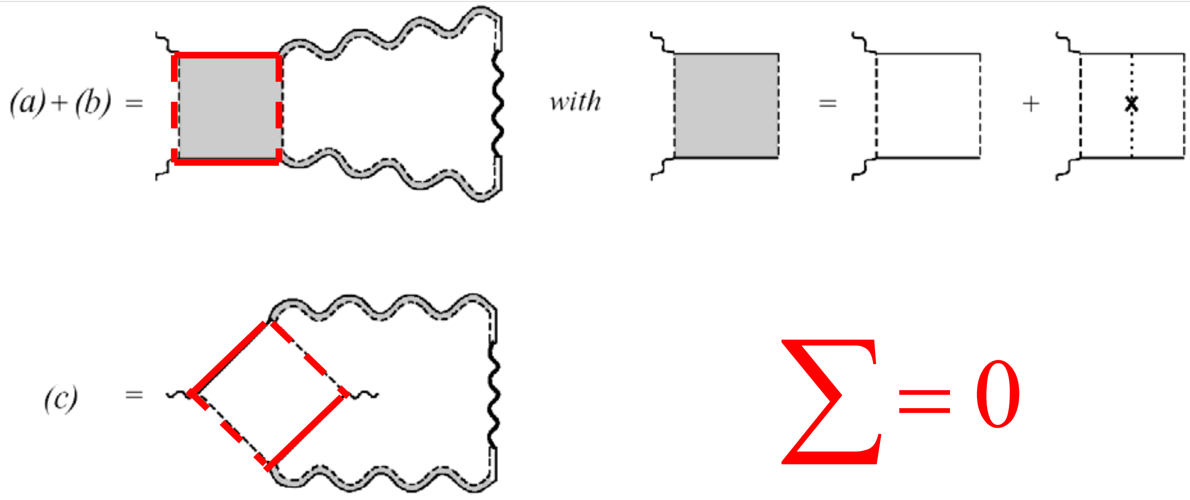
Disorder average: two amplitudes must follow the same trajectories
→ Diffusons

Exchange contribution

Altshuler,Aronov,Lee ’80





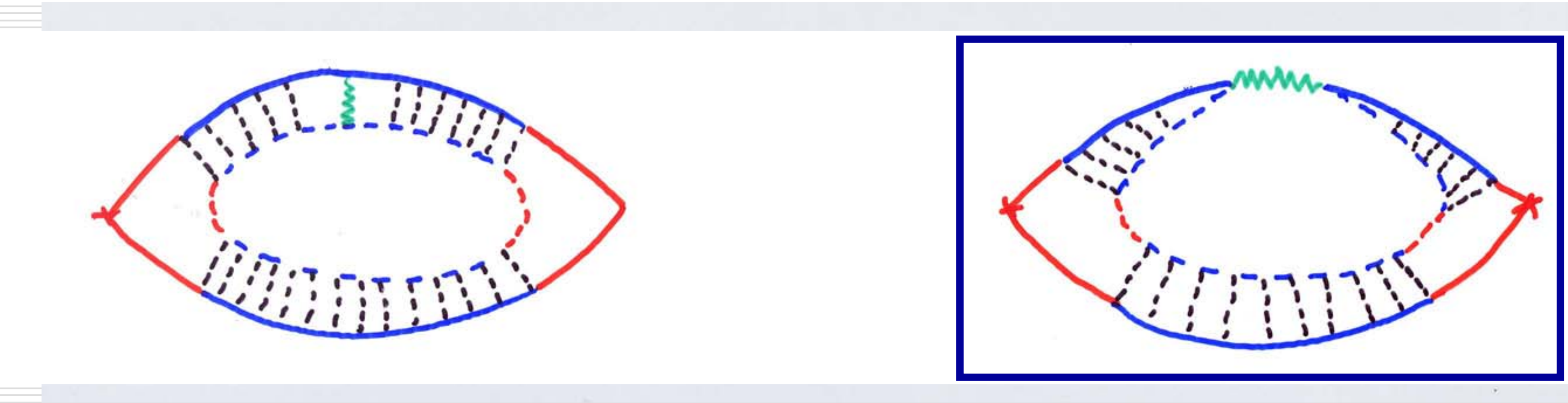


$\sum = 0$

Physical origin:

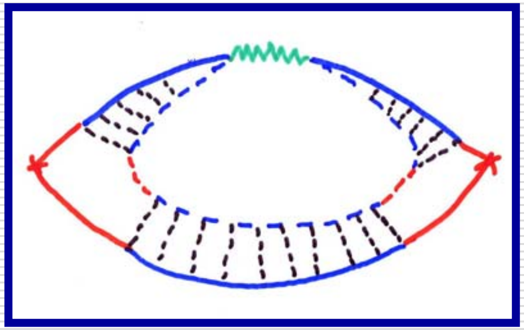
Direct (Hartree) Exchange (Fock)

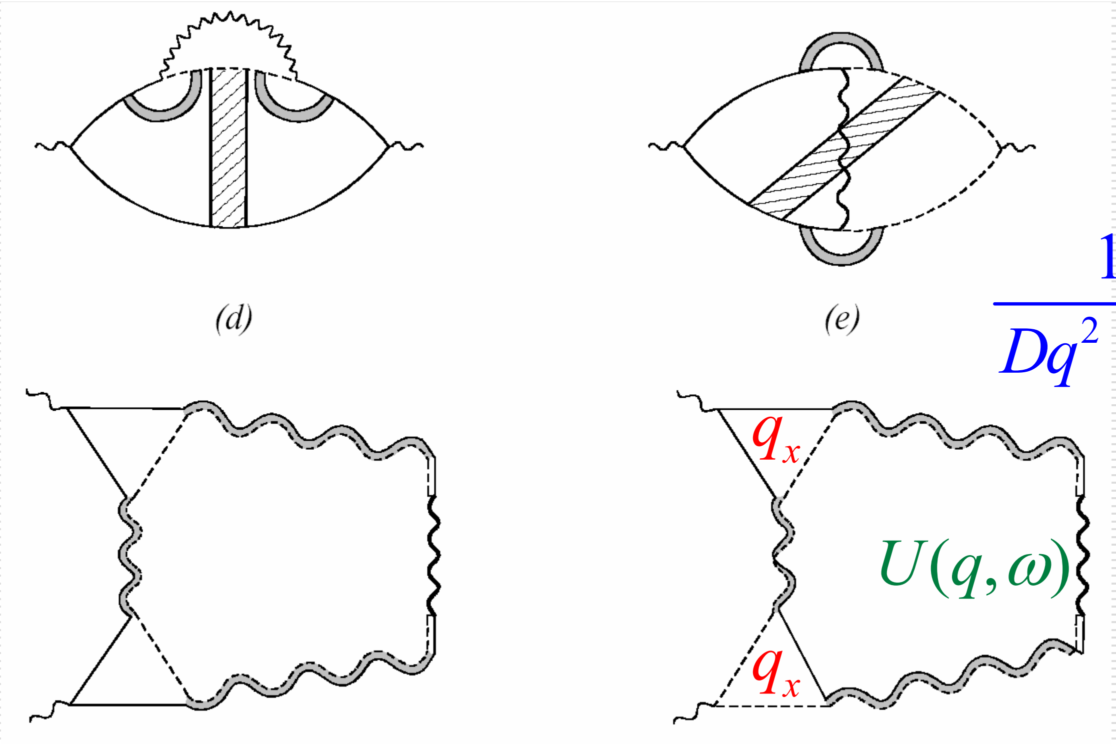
Disorder average: two amplitudes must follow the same trajectories
→ Diffusons



Exchange contribution

Altshuler,Aronov,Lee '80





$$\frac{1}{Dq^2 - i\omega}$$

$$\Delta\sigma_{ee}(T) = -\frac{2\sigma_0}{d\pi\Omega}\int_{-\infty}^{\infty} d\omega \frac{\partial}{\partial\omega}(\omega\coth\frac{\omega}{2T})\,\mathrm{Im}\sum_q Dq^2 \frac{U(q,\omega)}{(Dq^2 - i\omega)^3}$$

on a network ? → real space representation

Real space representation : example of weak localization

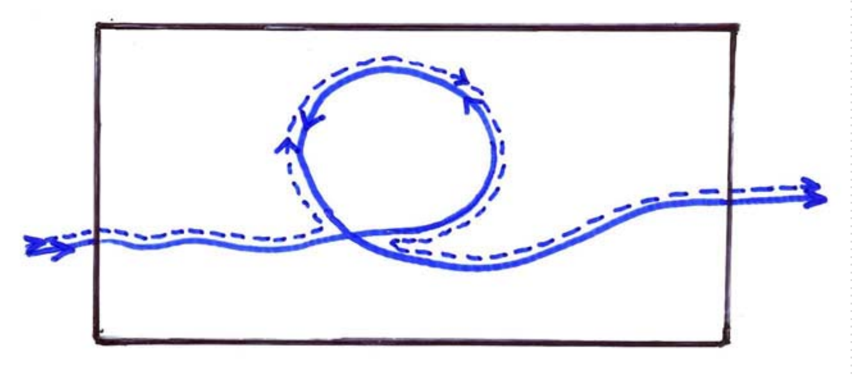
$$\Delta\sigma_{WL} = -2\frac{e^2 D}{\pi\hbar\Omega}\sum_q \frac{1}{Dq^2 + 1/\tau_\phi}$$

$$\Delta\sigma_{WL} = -\frac{2e^2 D}{\pi}\int_0^\infty P_c(t)e^{-t/\tau_\phi}dt$$

$P_c(t)$ **Cooperon contribution to the return probability**

$$P_c(\gamma) = \int_0^\infty P_c(t)e^{-\gamma t}dt$$

$$\gamma = \frac{1}{\tau_\phi}$$



$$P_c(\gamma) = \int_0^{\infty} P_c(t) e^{-\gamma t} dt$$

is solution of the diffusion equation

$$\left[\gamma - D\left(\nabla + i2e\frac{\vec{A}}{\hbar} \right)^2 \right] P_c(r, r', \gamma) = \delta(r - r')$$

$$P(\gamma) = \frac{1}{\Omega} \int P(r, r, \gamma) dr$$

Solve diffusion equation on a network

The spectral determinant

$$-D\,\Delta\psi = E_n\,\psi$$

$$\int P(t)\; e^{-\gamma t}\, dt = \sum_n \frac{1}{\gamma + E_n} = \frac{\partial}{\partial\gamma}\ln S(\gamma)$$

$$P(t) = \sum_n e^{-E_n t}$$

$$\gamma = \frac{1}{\tau_\phi} = \frac{D}{L_\phi^2}$$

$$S(\gamma) = \prod_n (\gamma + E_n)$$

Spectral determinant : contains all informations about diffusion

$$\int t^\alpha P(t) e^{-\gamma t} dt = \left(\frac{\partial}{\partial\gamma}\right)^{\alpha+1} \ln S(\gamma)$$

M. Pascaud, G.M., PRL 82, 4512 (1999)

Unified description of mesoscopic quantities

Weak-localization

$$\delta\sigma \propto \int P(t)e^{-t/\tau_\phi}dt$$

$$\delta\sigma \propto \frac{\partial}{\partial\gamma}\ln S(\gamma)$$

Average persistent current

$$\langle I_{ee}\rangle = -\frac{\lambda_0}{\pi}\frac{\partial}{\partial\phi}\int \frac{P_\phi(t)}{t^2}e^{-t/\tau_\phi}dt$$

$$\langle \delta\mathcal{M}_{ee}\rangle \propto \frac{\partial}{\partial B}\int_\gamma^\infty d\gamma' \ln S(\gamma', B)$$

Conductance fluctuations

$$\langle \delta\sigma^2\rangle \propto \int t\, P(t)e^{-t/\tau_\phi}dt$$

$$\delta\sigma^2 \propto \frac{\partial^2}{\partial\gamma^2}\ln S(\gamma)$$

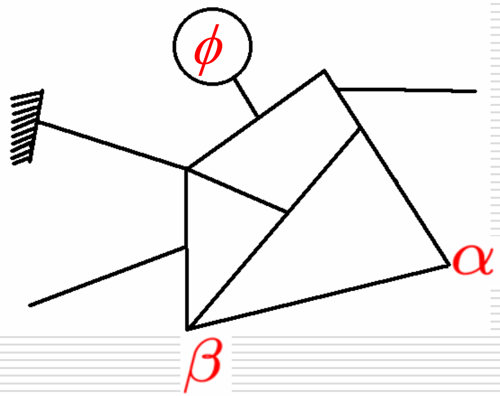
Variance of persistent current

$$\langle \delta I^2\rangle \propto \frac{\partial^2}{\partial\phi^2}\int \frac{P_\phi(t)}{t^3}e^{-t/\tau_\phi}dt$$

$$\langle \delta\mathcal{M}^2\rangle \propto \frac{\partial^2}{\partial B^2}\int_\gamma^\infty d\gamma'(\gamma-\gamma')\ln S(\gamma, B)$$

Spectral determinant on a network

Solve diffusion equation on each bond
Current conservation at the nodes



N nodes
N_B bonds

$$M_{\alpha\alpha} = \sum_{\beta} \coth \frac{L_{\alpha\beta}}{L_\phi}$$

$$M = N \times N \ \text{matrix}$$

$$\gamma = \frac{D}{L_\phi^2}$$

$$M_{\alpha\beta} = -\frac{e^{i\theta_{\alpha\beta}}}{\sinh \frac{L_{\alpha\beta}}{L_\phi}}$$

$$S(\gamma) = \left(\frac{L_\phi}{L}\right)^{N_B - N} \prod_{\alpha\beta} \sinh \frac{L_{\alpha\beta}}{L_\phi} \det M$$

$$\theta_{\alpha\beta} = \frac{4\pi}{\phi_0} \int_\alpha^\beta \vec{A} \bullet d\vec{l}$$

M. Pascaud, G.M., PRL 82, 4512 (1999)

cf: superconducting networks (Alexander, De Gennes)
scattering on 1D graphs (C. Texier, G.M.)

$$\frac{1}{L_\phi} \to ik \qquad \frac{1}{L_\phi} \to i\xi$$

Unified description of mesoscopic quantities

Weak-localization

$$\delta\sigma \propto \frac{\partial}{\partial\gamma} \ln S(\gamma)$$

Average persistent current

$$\langle \delta \mathcal{M}_{ee} \rangle \propto \frac{\partial}{\partial B} \int_{\gamma}^{\infty} d\gamma' \ln S(\gamma', B)$$

$$S(\gamma) = \left(\frac{L_\phi}{L} \right)^{N_B - N} \prod_{\alpha\beta} \sinh \frac{L_{\alpha\beta}}{L_\phi} \det M$$

Conductance fluctuations

$$\delta\sigma^2 \propto \frac{\partial^2}{\partial\gamma^2} \ln S(\gamma)$$

Variance of persistent current

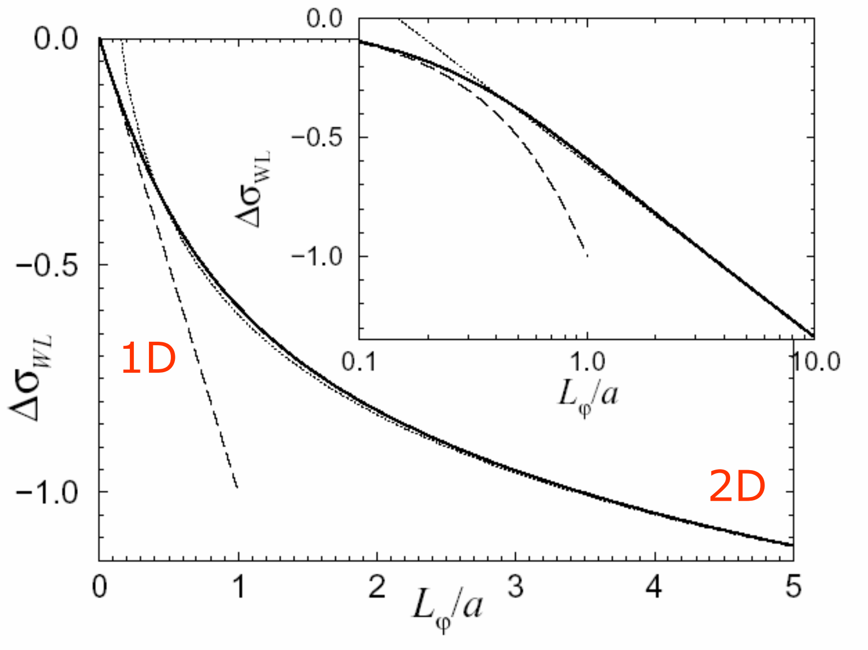
$$\langle \delta \mathcal{M}^2 \rangle \propto \frac{\partial^2}{\partial B^2} \int_{\gamma}^{\infty} d\gamma' (\gamma - \gamma') \ln S(\gamma, B)$$

Spectral determinant on the square lattice

Solve diffusion equation on each bond
Current conservation at the nodes → Tight binding problem on square lattice

$$S(\gamma) = \left(2\frac{L_\phi}{a}\sinh\frac{a}{L_\phi}\right)^{N_s} \prod_{k_x,k_y}\left(2\sinh a/L_\phi - \cos k_x a - \cos k_y a\right)$$

$$\Delta\sigma_{WL} = -\frac{2e^2 D}{\pi}\frac{\partial}{\partial\gamma}\ln S(\gamma)$$



Correction to the conductivity

$$\Delta\sigma_{ee}(T) = -\frac{2\sigma_0}{d\pi\Omega}\int_{-\infty}^{\infty} d\omega \frac{\partial}{\partial\omega}(\omega\coth\frac{\omega}{2T})\,\mathrm{Im}\sum_q Dq^2 \frac{U(q,\omega)}{(Dq^2 - i\omega)^3}$$

in terms of $P_d(t)$ classical return probability

$$P_d(\gamma) = \int_0^{\infty} P_d(t)e^{-\gamma t}dt$$

Correction to the conductivity

$$\Delta\sigma_{ee} = -\frac{2\sigma_0}{d\pi\Omega}\int_{-\infty}^{\infty} d\omega \frac{\partial}{\partial\omega}\left(\omega\coth\frac{\omega}{2T}\right)\mathrm{Im}\sum_q Dq^2 \frac{U(q,\omega)}{(Dq^2 - i\omega)^3}$$

$$U(q,\omega) = \frac{1}{\nu}\frac{Dq^2 - i\omega}{Dq^2}$$

Dynamically screened interaction

$$\Delta\sigma_{ee} = -\frac{2e^2 D}{d\pi\Omega}\int_{-\infty}^{\infty} d\omega \frac{\partial^2}{\partial\omega^2}\left(\omega\coth\frac{\omega}{2T}\right)\mathrm{Re}\sum_q \frac{1}{(Dq^2 - i\omega)}$$

Correction to the conductivity

$$\Delta\sigma_{ee} = -\frac{2e^2 D}{d\pi\Omega}\int_{-\infty}^{\infty} d\omega \frac{\partial^2}{\partial\omega^2}\left(\omega\coth\frac{\omega}{2T}\right)\mathrm{Re}\sum_d \frac{1}{(Dq^2 - i\omega)}$$

$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2 D}{\pi}\int_0^{\infty} dt \left(\frac{\pi T t}{\sinh \pi T t}\right)^2 P_d(t)$$

$$\lambda_\sigma \approx \frac{4}{d} - \frac{3F}{2}$$

$$F = \frac{\langle U(p-p')\rangle}{U(0)}$$

WL correction vs. e-e correction

$$\Delta\sigma_{WL} = -\frac{2e^2 D}{\pi}\int_0^\infty dt\, e^{-t/\tau_\phi} P_c(t)$$

$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2 D}{\pi}\int_0^\infty dt \left(\frac{\pi T t}{\sinh \pi T t}\right)^2 P_d(t)$$

$$\left(\frac{\pi T t}{\sinh \pi T t}\right)^2 = 4(\pi T t)^2 \sum_1^\infty n\ e^{-2n\pi T t}$$

$$\gamma = \frac{1}{\tau_\phi}$$

$$\gamma_n = 2n\pi T$$

$$\Delta\sigma_{WL} = -\frac{2e^2 D}{\pi}\int_0^\infty dt e^{-\gamma t} P_c(t)$$

$$\Delta\sigma_{ee} = -4\lambda_\sigma \frac{e^2 D}{\pi}\sum_1^\infty n \int_0^\infty dt\ (\pi T t)^2 e^{-\gamma_n t} P_d(t)$$

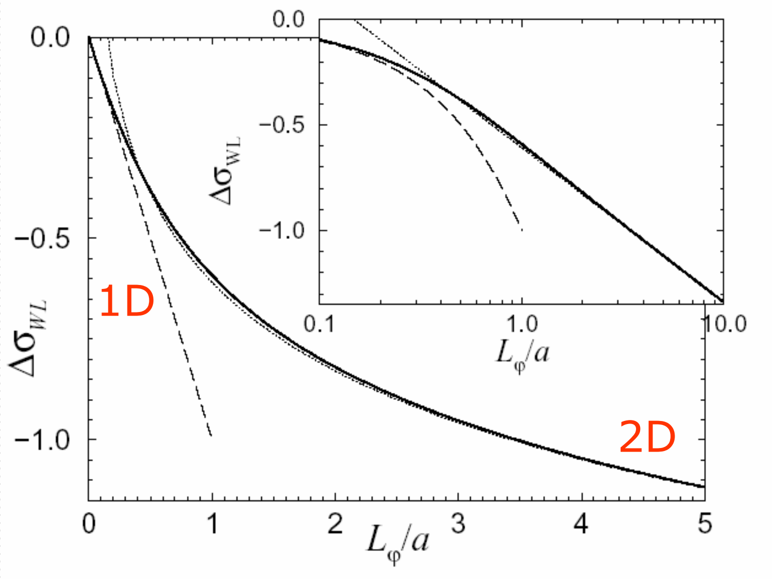
WL correction vs. e-e correction

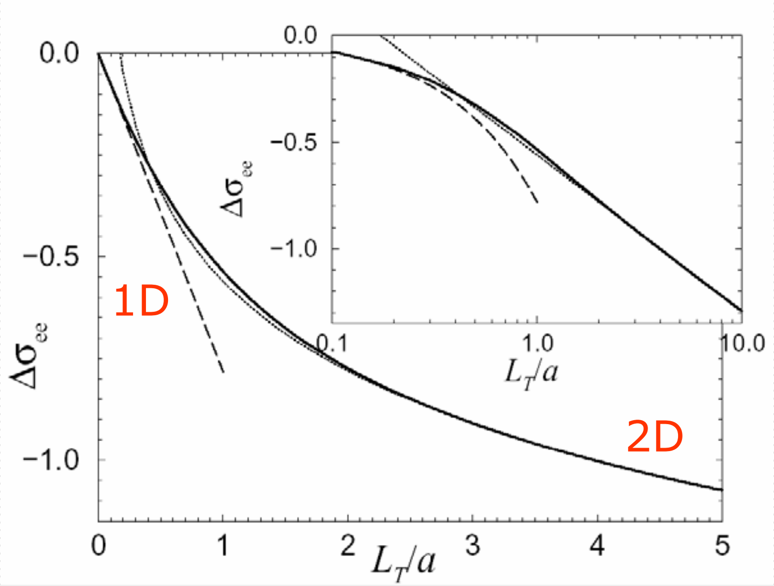
$$\Delta\sigma_{WL} = -\frac{2e^2}{\pi\Omega}\frac{\partial}{\partial\gamma}\ln S(\gamma)$$

$$\Delta\sigma_{ee} = -4\lambda_\sigma \frac{e^2}{\pi\Omega}\sum_1^\infty \frac{\gamma_n^2}{n}\frac{\partial^3}{\partial\gamma_n^3}\ln S(\gamma_n)$$

$$\Delta\sigma_{WL} = -2\frac{e^2}{h}a\,F_{WL}\left(L_\phi/a\right)$$

$$\Delta\sigma_{ee} = -\lambda_\sigma\frac{e^2}{h}a\,F_{ee}\left(L_T/a\right)$$





WL correction vs. e-e correction

$$L_\phi \ll a \qquad\qquad L_T \ll a$$

$$\Delta\sigma_{WL} = -2\frac{e^2}{h}a\left(L_\phi - \frac{L_\phi^2}{2a} + \ldots\right) \qquad \Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{h}\left(0.78L_T - \frac{\pi}{12}\frac{L_T^2}{a}\right)$$

To be compared with the case of a wire of length a:

$$\Delta\sigma_{WL} = -2\frac{e^2}{h}a\left(L_\phi - \frac{L_\phi^2}{a} + \ldots\right) \qquad \Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{h}\left(0.78L_T - \frac{\pi}{6}\frac{L_T^2}{a}\right)$$

More generally, for coordinence z

$$\Delta\sigma_{WL} = -2\frac{e^2}{h}a\left(L_\phi - \frac{L_\phi^2}{(z/2)a} + \ldots\right) \qquad \Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{h}\left(0.78L_T - \frac{\pi}{6}\frac{L_T^2}{(z/2)a}\right)$$

$$L_\phi \ll a$$

$$L_T \ll a$$

$$\Delta\sigma_{WL} = -2\frac{e^2}{h}a\left(L_\phi - \frac{L_\phi^2}{(z/2)a} + \ldots\right)$$

$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{h}\left(0.78L_T - \frac{\pi}{6}\frac{L_T^2}{(z/2)a}\right)$$

$z = 4$

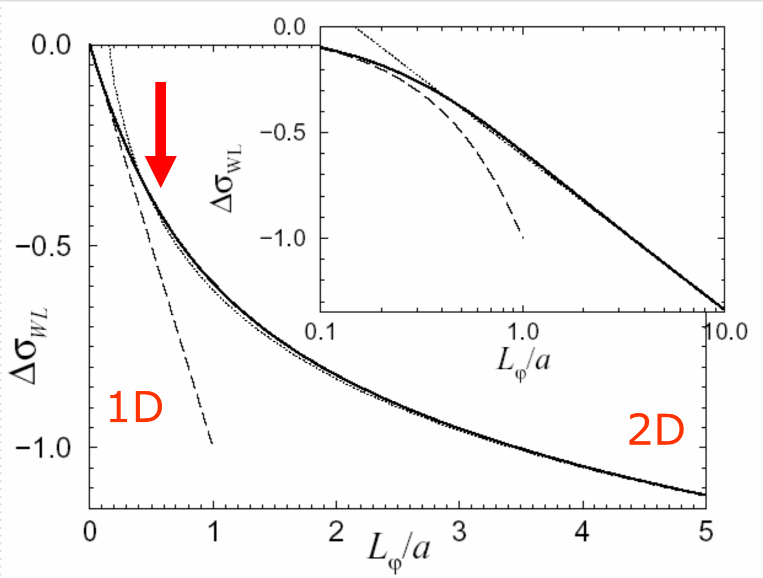
$z = 2$

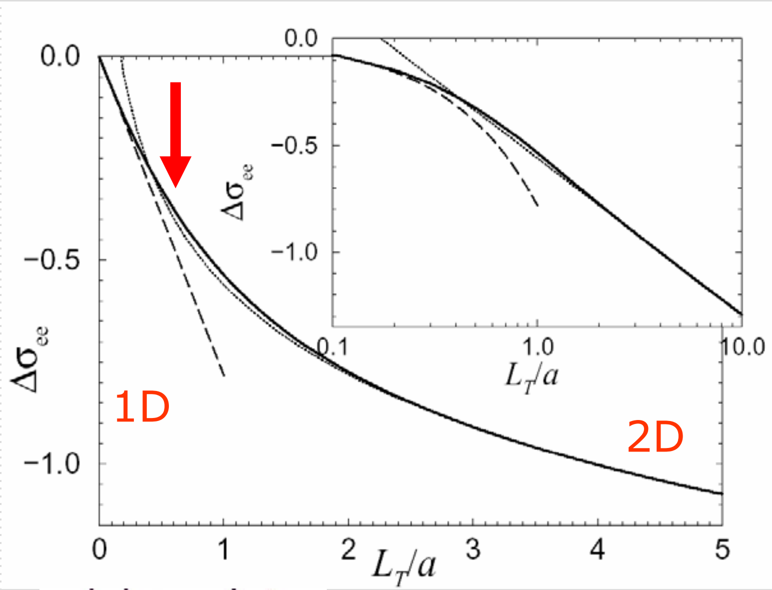
$L_\phi \gg a$

$L_T \gg a$

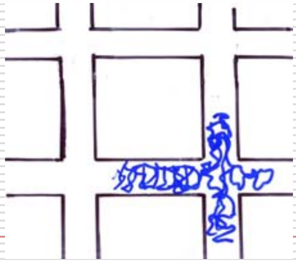
$$\Delta\sigma_{WL} = -2\frac{e^2}{\pi h}a\left(\ln\frac{L_\phi}{a} + 1.91 + \ldots\right)$$

$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{\pi h}a\left(\ln\frac{L_T}{a} + 1.76 + \ldots\right)$$



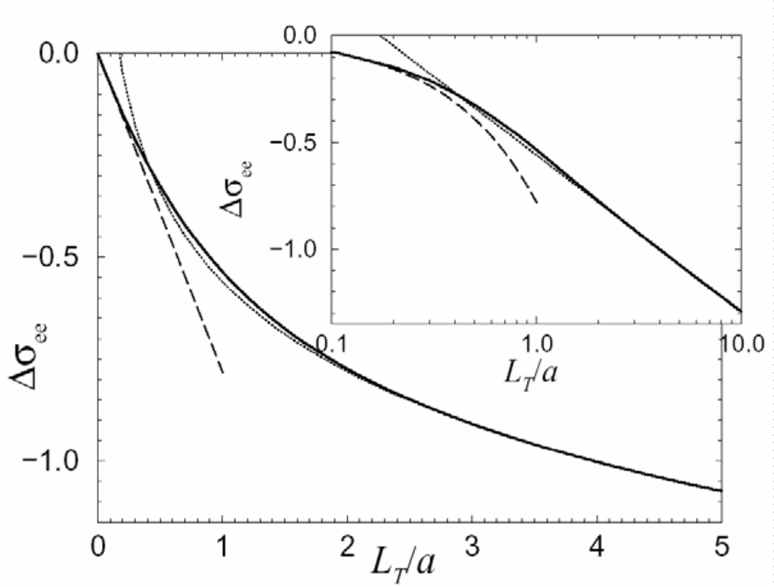


Cross-over $L_\phi, L_T \approx a$

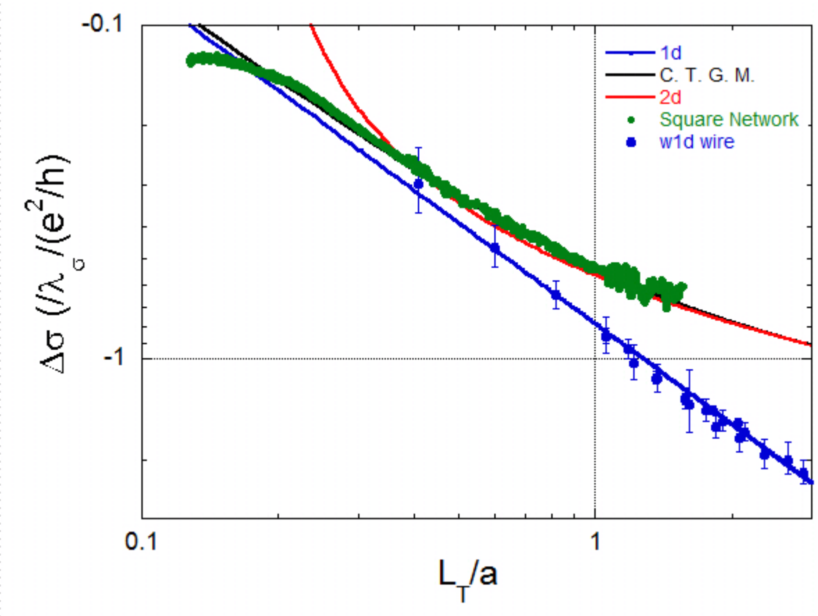


Experiments

Bauerle,Mallet,Mailly,Saminadayar,Schopfer, GRENOBLE



Network of silver wires



$$\Delta\sigma_{ee} = -\lambda_\sigma \frac{e^2}{h} a\, F_{ee}\left(L_T / a\right)$$

$$\lambda_\sigma(network) \approx 3.1 - 3.5$$

$$\lambda_\sigma(wire) \approx 3.2$$

Silver

$$k_F^{-1} = 0.083nm$$

$$\kappa^{-1} = 0.055nm$$

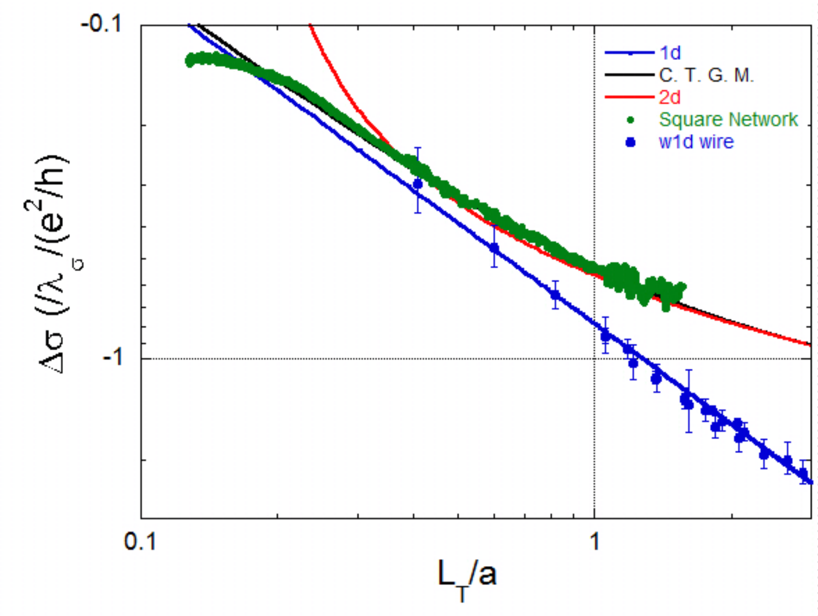
$$F(\kappa / k_F) \approx 0.58$$

$$\lambda_\sigma \approx \frac{4}{d} - \frac{3F}{2}$$

$$\lambda_\sigma^{th}(1d) \approx 3.2$$

~~$\lambda_d^{th}(2d) \approx 1.2$~~

Network of silver wires



$$\lambda_\sigma(network) \approx 3.1 - 3.5$$

$$\lambda_\sigma(wire) \approx 3.2$$

Experiments and conclusion

Silver

$$k_F^{-1} = 0.083nm$$

$$\kappa^{-1} = 0.055nm$$

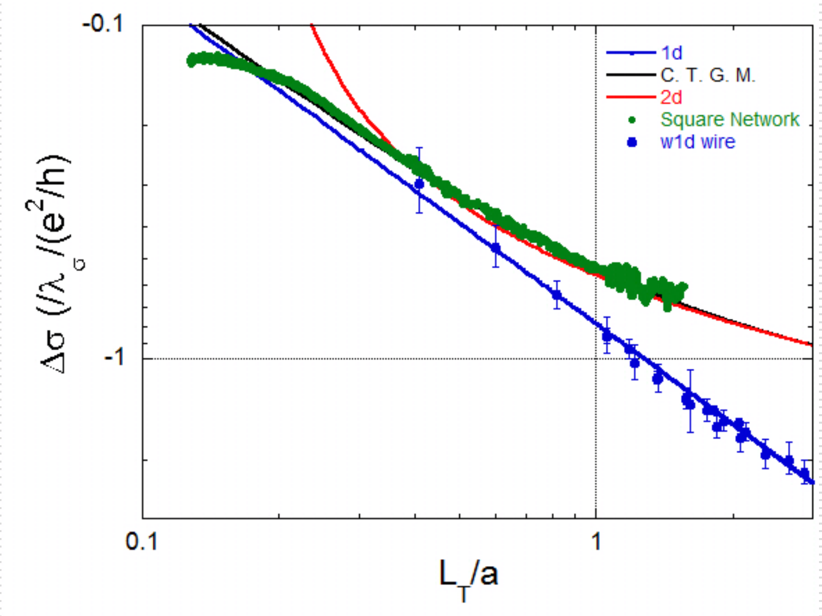
$$F(\kappa / k_F) \approx 0.58$$

$$\lambda_\sigma \approx \frac{4}{d} - \frac{3F}{2}$$

$$\lambda_\sigma^{th}(1d) \approx 3.2$$

~~$\lambda_d^{th}(2d) \approx 1.2$~~

Network of silver wires



$$\lambda_\sigma(network) \approx 3.1 - 3.5$$

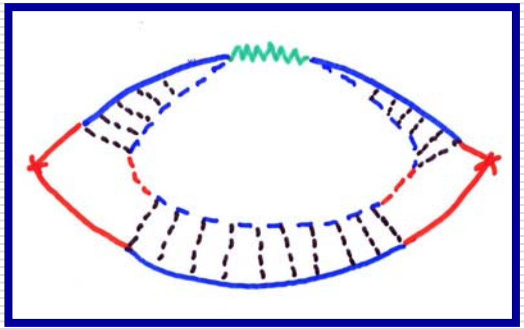
$$\lambda_\sigma(wire) \approx 3.2$$

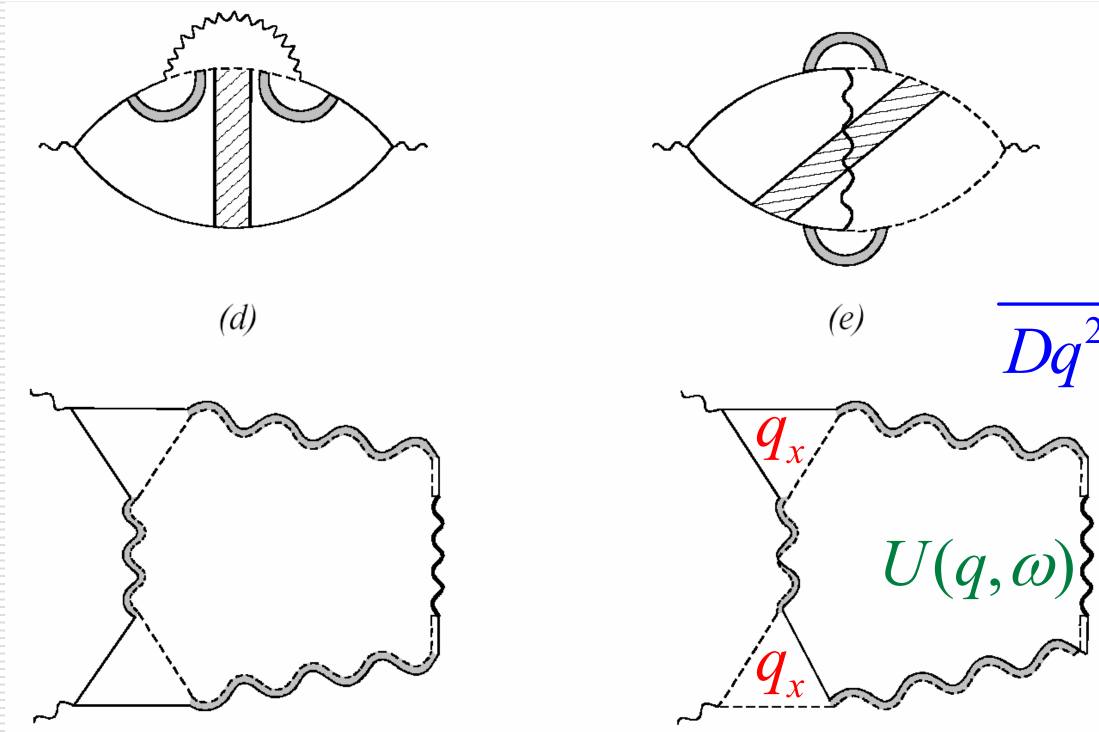
explains cross-over 1d (high T) to 2d (low T) behaviour

Probes 1d diffusion in wire and 1d-2d diffusion on the network

Exchange contribution

Altshuler,Aronov,Lee '80





$$\frac{1}{Dq^2 - i\omega}$$

$$\Delta\sigma_{ee}(T) = -\frac{2\sigma_0}{d\pi\Omega}\int_{-\infty}^{\infty} d\omega \frac{\partial}{\partial\omega}\left(\omega\coth\frac{\omega}{2T}\right)\mathrm{Im}\sum_q Dq^2 \frac{U(q,\omega)}{(Dq^2 - i\omega)^3}$$

$$\lambda_\sigma \approx \frac{4}{d} - \frac{3F}{2}$$

Finkelshtein '83

$$\lambda_\sigma = \frac{4}{d} + \frac{48}{d(d-2)F}\left[1 + \frac{dF}{4} - (1 + \frac{F}{2})^{d/2}\right] \approx \frac{4}{d} - \frac{3F}{2} + \frac{4-d}{8}F^2$$