Gilles Montambaux, Université Paris-Sud, CNRS, Orsay (France)

Cours 4

Fluctuations universelles de conductance

Antilocalisation et couplage spin-orbite

Processus de déphasage

Interactions e-e

8 septembre 2012

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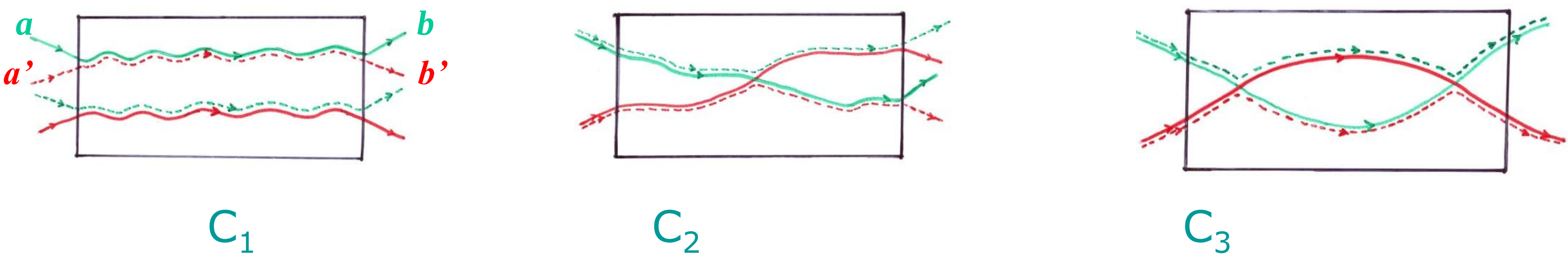
Fluctuations de speckle *vs* fluctuations de conductance — résumé



$$\overline{\delta T_{ab}\delta T_{a'b'}} = \overline{T_{ab}}\ \overline{T_{a'b'}}\left(\delta_{a,a'}\delta_{b,b'} + \frac{2}{3g}\left[\delta_{a,a'} + \delta_{b,b'}\right] + \frac{2}{15g^2}\right)$$

$$g = \sum_{a,b} T_{ab}$$

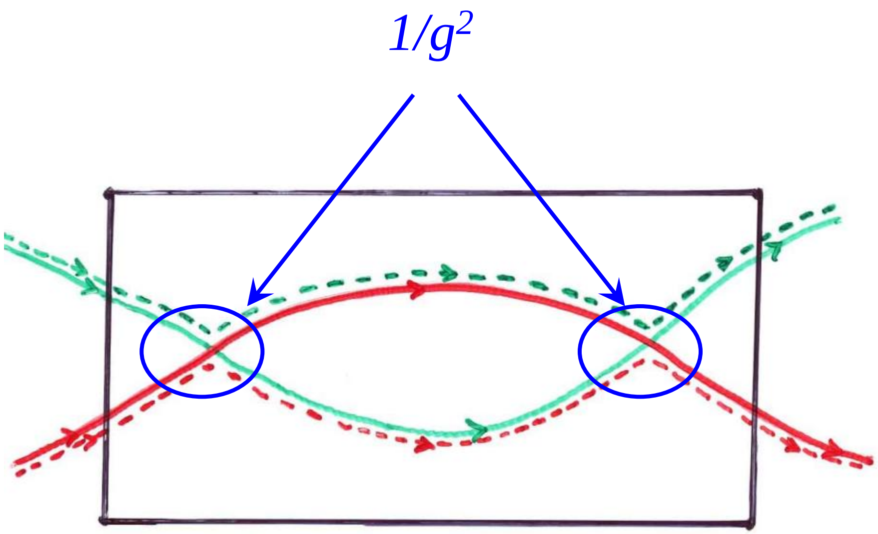
$$\overline{\delta g^2} = \sum_{a,b,a',b'} \overline{\delta T_{ab}\delta T_{a'b'}}$$

$$\overline{\delta g^2} = \frac{g^2}{M^2} + \frac{4}{3}\frac{g}{M} + \frac{2}{15}$$

$$\frac{g}{M} \sim \frac{l_e}{L} \ll 1$$

Fluctuations universelles de conductance

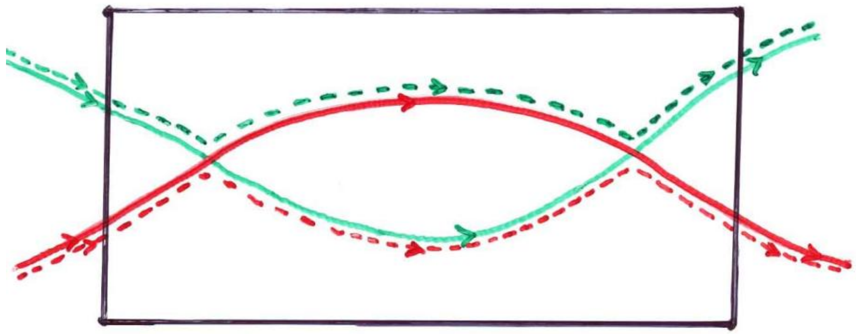
Fluctuations universelles de conductance



Fluctuations de conductance = 2 conductances + 2 croisements

$$\overline{\delta G^2} = \left(\frac{e^2}{h}\right)^2 g^2 \times \frac{1}{g^2} \Rightarrow \left(\frac{e^2}{h}\right)^2$$

Fluctuations universelles de conductance



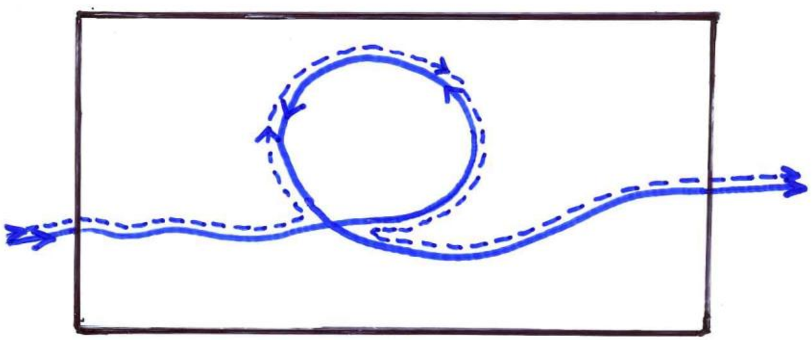
Distribution de boucles

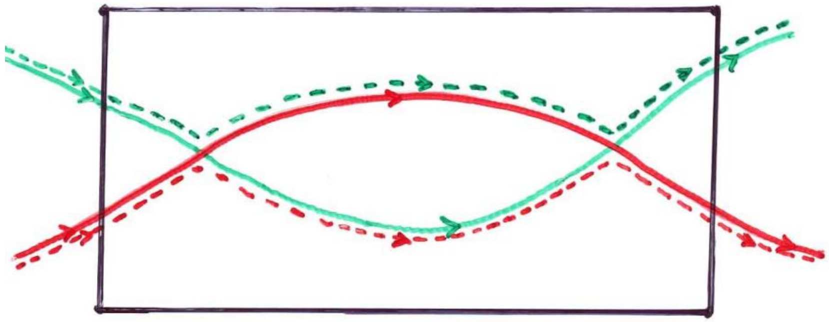
$$\overline{\delta G^2} \sim \left(\frac{e^2}{h}\right)^2 g^2 \times \frac{1}{g^2} \times \langle t\, P(t) \rangle = \left(\frac{e^2}{h}\right)^2 g^2 \times \frac{1}{g^2} \times \int_{\tau_e}^{\min(\tau_D,\tau_\phi,\tau_B)} t\, P(t) \frac{dt}{\tau_D^2}$$

$$P(t) = P_{cl}(t) + P_{\mathrm{int}}(t)$$

$$\overline{\delta G^2} = 12 \left(\frac{2e^2}{h}\right)^2 \int_0^\infty t\, P(t) \frac{dt}{\tau_D^2}$$

Why are the fluctuations universal and weak localization is not ?





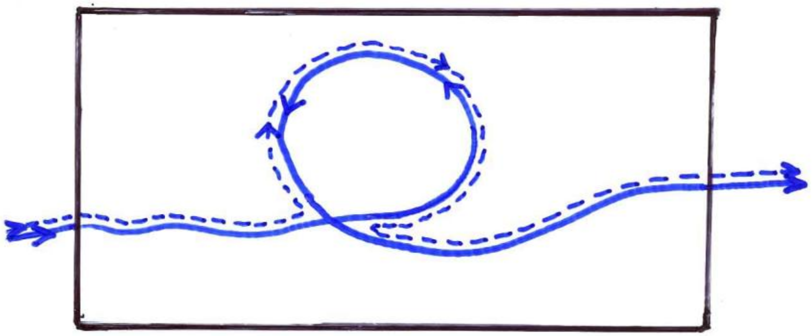
$$\frac{\Delta G}{G_{cl}} \sim \frac{1}{g} \int_{\tau_e}^{\tau_D} P(t) \frac{dt}{\tau_D}$$

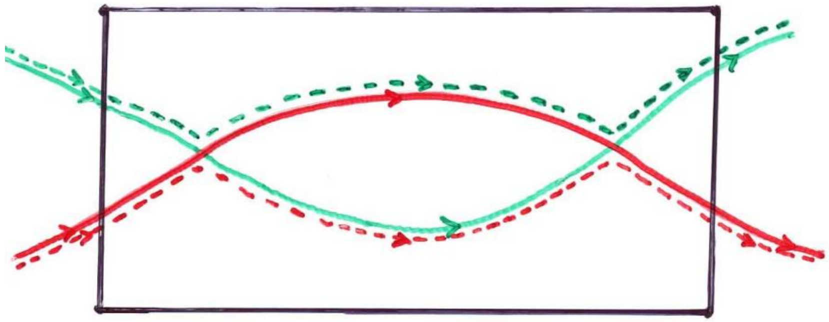
$$\frac{\left\langle \delta G^2 \right\rangle}{G_{cl}^2} \sim \frac{1}{g^2} \int_{\tau_e}^{\tau_D} t\, P(t) \frac{dt}{\tau_D^2}$$

$$P(t) = \left(\frac{\tau_D}{4\pi t} \right)^{d/2}$$

Why are the fluctuations universal and weak localization is not ?

$$g = \frac{G}{2e^2/h}$$





$$\Delta g \sim \int_{\tau_e}^{\tau_D} P(t) \frac{dt}{\tau_D}$$

$$\langle \delta g^2 \rangle \sim \int_{\tau_e}^{\tau_D} t\, P(t) \frac{dt}{\tau_D^2}$$

$$P(t) = \left(\frac{\tau_D}{4\pi t} \right)^{d/2}$$

$$\rightarrow \int_{\tau_e/\tau_D}^{1} \frac{dx}{x^{d/2}}$$ Universal if d < 2

$$\rightarrow \int_{\tau_e/\tau_D}^{1} \frac{dx}{x^{d/2-1}}$$ Universal if d < 4

Exemple: le fil quasi-1D $L \ll L_\phi$ limite mésoscopique

← WL UCF →

$$P(t) = \sum e^{-Dq^2 t}$$

$$q = \frac{n\pi}{L} \qquad n = 1, 2, \ldots, \infty$$

WL:

$$\overline{\Delta g} = -2 \int_0^\infty P(t) \frac{dt}{\tau_D}$$

$$\overline{\Delta g} = -2 \sum \frac{1}{(qL)^2}$$

$$\overline{\Delta g} = -\frac{2}{\pi^2} \sum \frac{1}{n^2}$$

$$\overline{\Delta g} = -\frac{1}{3}$$

UCF:

$$\overline{\delta g^2} = 12 \int_0^\infty t\, P(t) \frac{dt}{\tau_D^2}$$

$$\overline{\delta g^2} = 12 \sum \frac{1}{(qL)^4}$$

$$\overline{\delta g^2} = \frac{12}{\pi^4} \sum \frac{1}{n^4}$$

$$\overline{\delta g^2} = \frac{2}{15}$$

Exemple: le fil quasi-1D $\quad L / L_\phi \quad$ quelconque

$$\overline{\Delta g} = -2 \int_0^\infty P(t) \frac{dt}{\tau_D}$$

← WL $\qquad$ UCF →

$$\overline{\delta g^2} = 12 \int_0^\infty t\, P(t) \frac{dt}{\tau_D^2}$$

$$P(t) = \sum e^{-Dq^2 t} e^{-t/\tau_\phi}$$

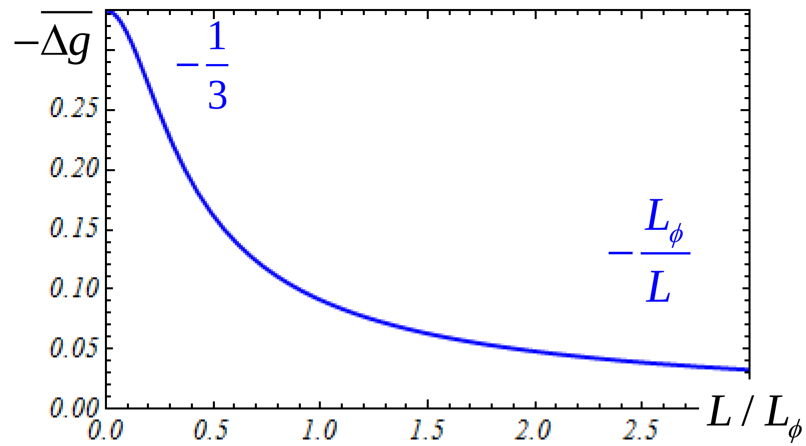
$$\overline{\Delta g} = -2 \sum \frac{1}{(qL)^2 + \left(L/L_\phi\right)^2}$$

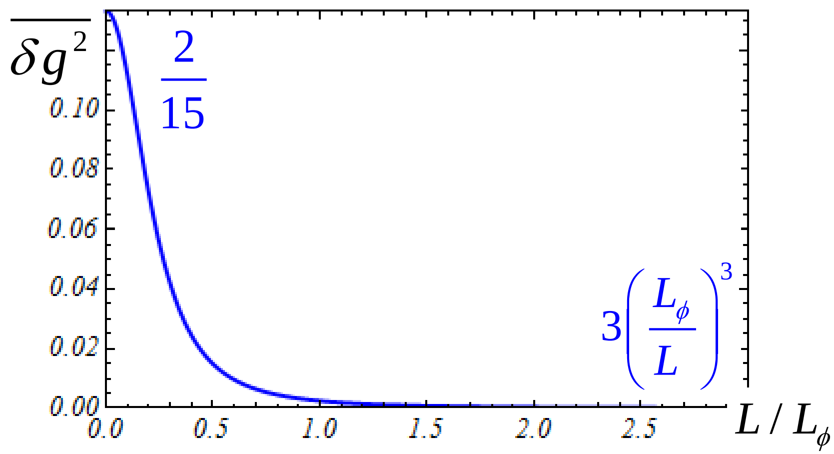
$$\overline{\delta g^2} = 12 \sum \frac{1}{\left((qL)^2 + \left(L/L_\phi\right)^2\right)^2}$$

$$q = \frac{n\pi}{L} \qquad n = 1, 2, \ldots, \infty$$

$$\overline{\Delta g} = -\frac{2}{\pi^2} \sum \frac{1}{n^2 + \left(\pi L/L_\phi\right)^2}$$

$$\overline{\delta g^2} = \frac{12}{\pi^4} \sum \frac{1}{\left(n^2 + \left(\pi L/L_\phi\right)^2\right)^2}$$





limite macroscopique $L \gg L_\phi$

$$\overline{\delta g^2} = 12\int_0^\infty t\left(\frac{\tau_D}{4\pi t}\right)^{d/2} e^{-t/\tau_\phi}\,\frac{dt}{\tau_D^2}$$

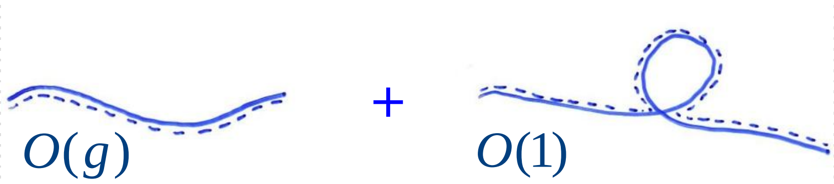
$$P(t) = \left(\frac{\tau_D}{4\pi t}\right)^{d/2}$$

$$\boxed{\overline{\delta g^2} \sim \left(\frac{\tau_\phi}{\tau_D}\right)^{\frac{4-d}{2}} = \left(\frac{L_\phi}{L}\right)^{4-d}}$$

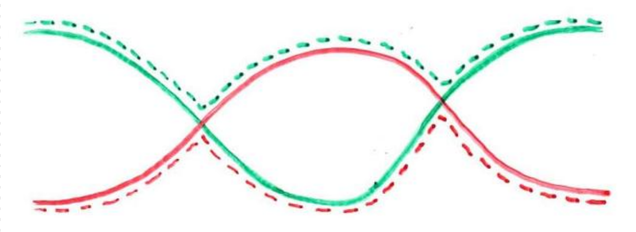
* addition classique de N éléments incohérents : $N = \left(\frac{L}{L_c}\right)^d \longrightarrow \delta G \sim \frac{1}{L^{(4-d)/2}}$

Gaussian fluctuations ?

$$\overline{G} =$$



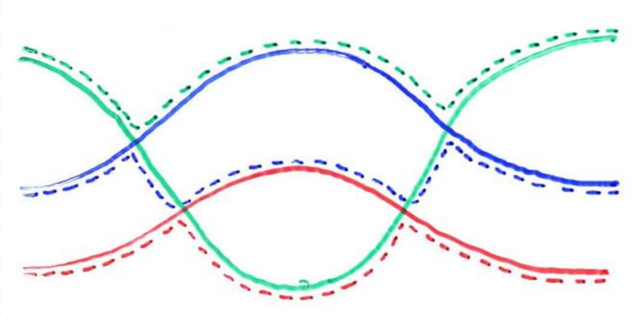
$$\overline{\delta G^2} =$$



2 conductances and 2 crossings

$$g^2 \times \frac{1}{g^2} \sim O(1)$$

$$\overline{\delta G^3} =$$



3 conductances and 4 crossings

$$g^3 \times \frac{1}{g^4} \sim O\left(\frac{1}{g}\right)$$

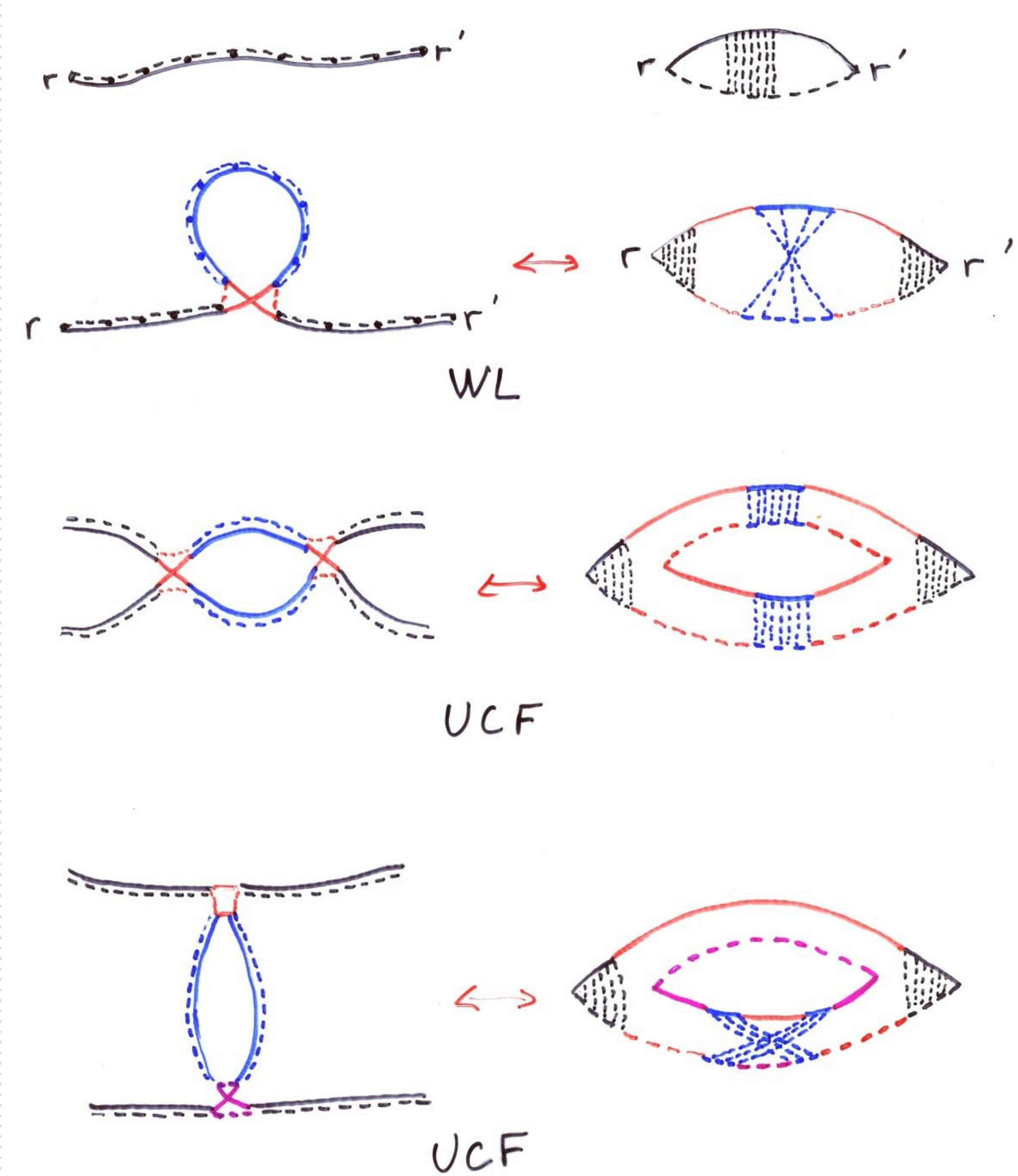
$$\overline{\delta G^n} =$$

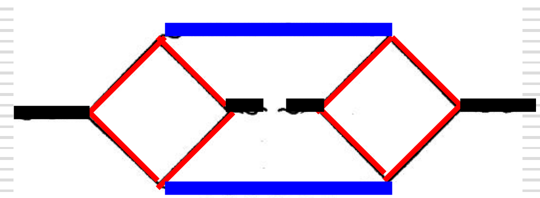
n conductances and (2n- 2) crossings

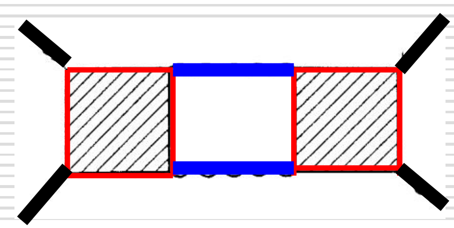
$$\overline{\delta G^n} \sim g^n \times \frac{1}{g^{2n-2}} \sim O\left(\frac{1}{g^{n-2}}\right)$$

Gaussian fluctuations in the limit $g \rightarrow \infty$

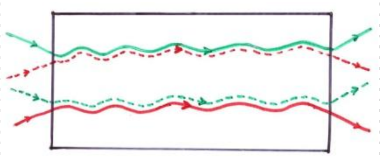
« Real diagrams »



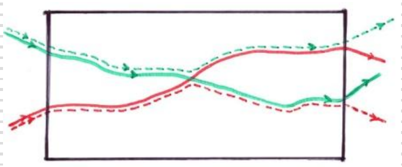




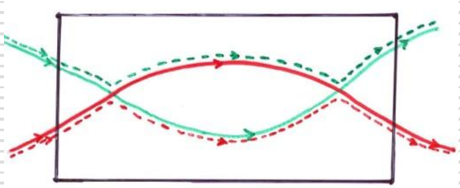
Speckle contribution C_2 and shot noise: any connection?



C_1



C_2



C_3

$$\overline{\delta T_{ab}\delta T_{a'b'}} = \overline{T_{ab}}\ \overline{T_{a'b'}} \left(f(a,a',b,b') + \frac{2}{3g}\left[F(a,a') + F(b,b')\right] + \frac{2}{15g^2} \right)$$

Shot noise
in a diffusive wire

$$S = 2eI \times \frac{1}{3}$$

Fano factor

$$S = 2\frac{e^2}{h}eV\, T(1-T)$$

1 Canal

$$S = 2\frac{e^2}{h}eV\, Tr\, tt^\dagger(1-tt^\dagger)$$

Multicanal

Shot noise in a diffusive conductor ←→ C_2 speckle

$$G = \frac{e^2}{h} Tr\, tt^\dagger$$

$$S = 2\frac{e^2}{h} eV\, Tr\, tt^\dagger (1 - tt^\dagger)$$

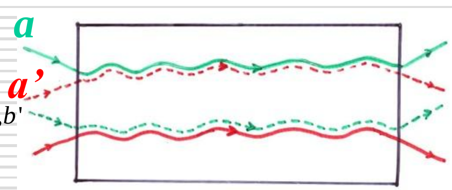
$$\langle \delta g^2 \rangle = \left\langle \left(Tr\, tt^\dagger \right)^2 \right\rangle - \left\langle Tr\, tt^\dagger \right\rangle^2 = \frac{2}{15}$$

$$\left\langle Tr\, tt^\dagger tt^\dagger \right\rangle ?$$

$$\left\langle \sum_{aba'b'} t_{ab} t^\dagger_{ba}\; t_{a'b'} t^\dagger_{b'a'} \right\rangle$$

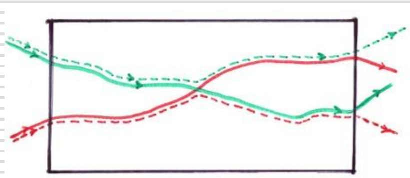
$$\left\langle \sum_{aba'b'} t_{ab} t^\dagger_{ba'}\; t_{a'b'} t^\dagger_{b'a} \right\rangle$$

$g^2 \delta_{a,a'}\, \delta_{b,b'}$



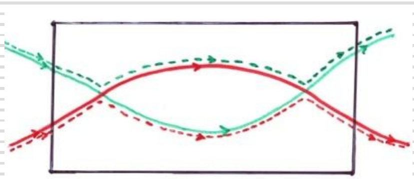
→ 0

$\frac{2}{3} g\, \delta_{b,b'}$

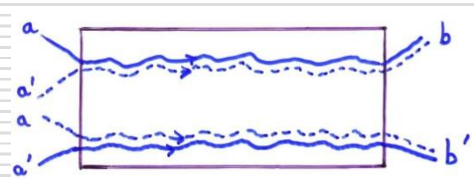


→ 0

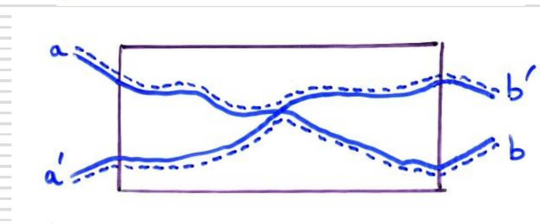
$\frac{2}{15}$



→ $\frac{2}{15}$



→ 0



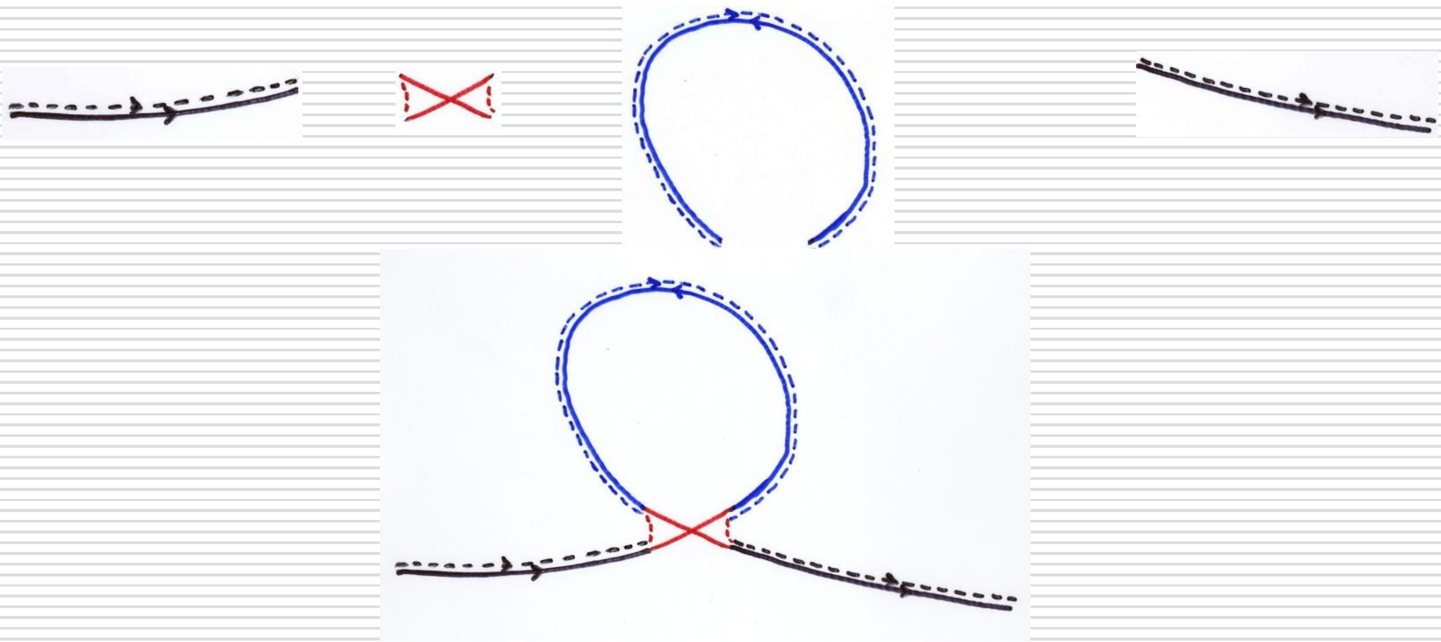
→ $g^2 \times \frac{2}{3g}$

$$S = 2\frac{e^2}{h} eV\, g \left(1 - \frac{2}{3} \right) = 2eI \times \frac{1}{3}$$

Conclusion

Quantum transport of electrons and light in diffusive systems

« Lego » Classical diffusion (diffuson or cooperon)
Quantum crossings



Simple formulation of phase coherent properties in the limit g>>1

Couplage spin-orbite

Nouveau temps caractéristique

$$\tau_{so}$$

Temps moyen sur lequel un électron perd la direction de son spin initial

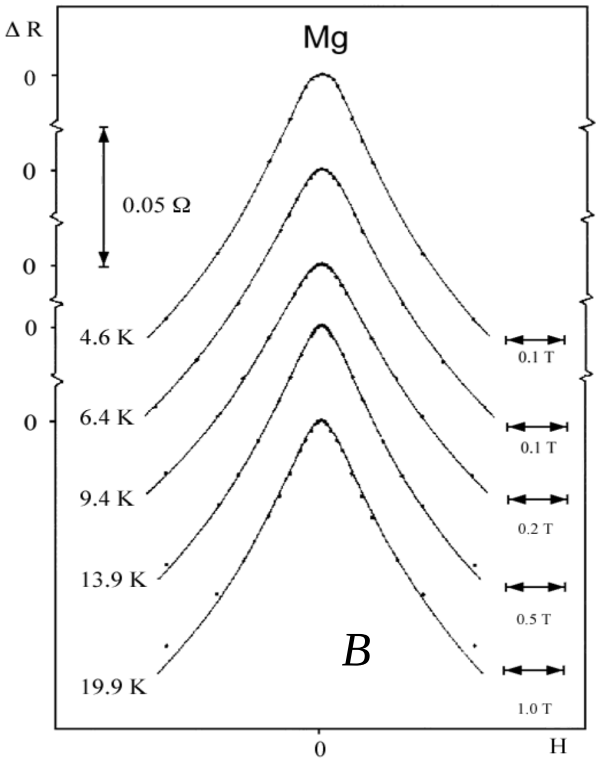
« Antilocalisation »

Hikami, Larkin and Nagaoka,
Spin-Orbit interaction and magnetoresitance in a two-dimensional random system
Prog. Theor. Phys. 63,707 (1980)

No spin-orbit coupling

Bergmann, 84

δR



WL

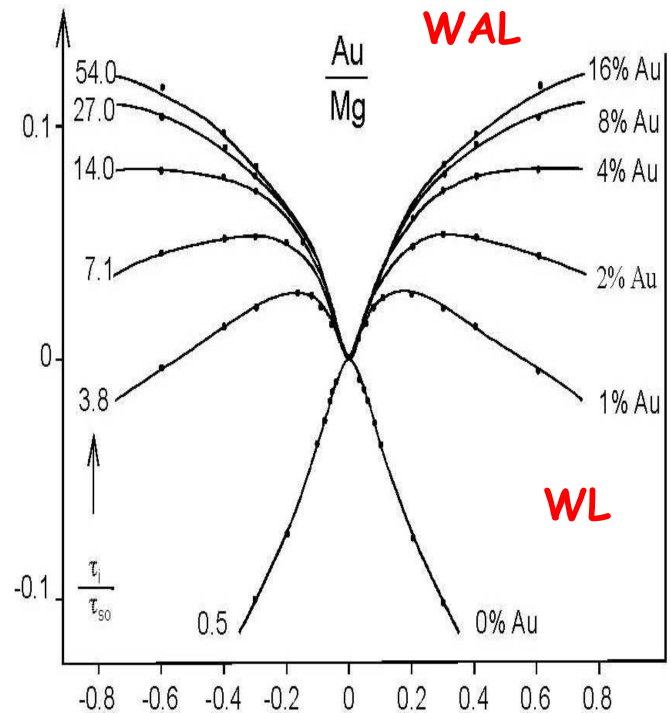
Positive magnetoconductance

weak-localization

Spin-orbit coupling

Bergmann, 84

δR



WAL

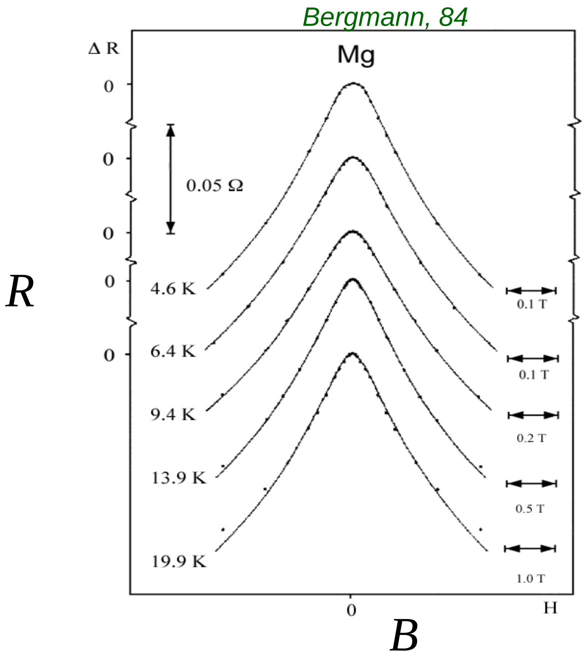
WL

Negative magnetoconductance

weak-antilocalization

Example 1: weak localization in 2 D

$$\Delta G(B) = -4\frac{e^2}{h}\int_0^\infty \frac{B/\phi_0}{\sinh 4\pi BDt/\phi_0}\left(e^{-t/\tau_\phi} - e^{-t/\tau_e}\right)\frac{dt}{\tau_D}$$



Correction de localisation faible

$$\Delta G(B) = -\frac{e^2}{\pi h}\left[\psi\left(\frac{1}{2}+\frac{\hbar}{4eBD\tau_e}\right) - \psi\left(\frac{1}{2}+\frac{\hbar}{4eBD\tau_\phi}\right)\right]$$

Magnétorésistance

$$\delta G(B) = \Delta G(B) - \Delta G(0)$$

$$\delta G = \frac{e^2}{\pi h}\left[\ln\left(\frac{4eBD\tau_\phi}{\hbar}\right) + \psi\left(\frac{1}{2}+\frac{\hbar}{4eBD\tau_\phi}\right)\right]$$

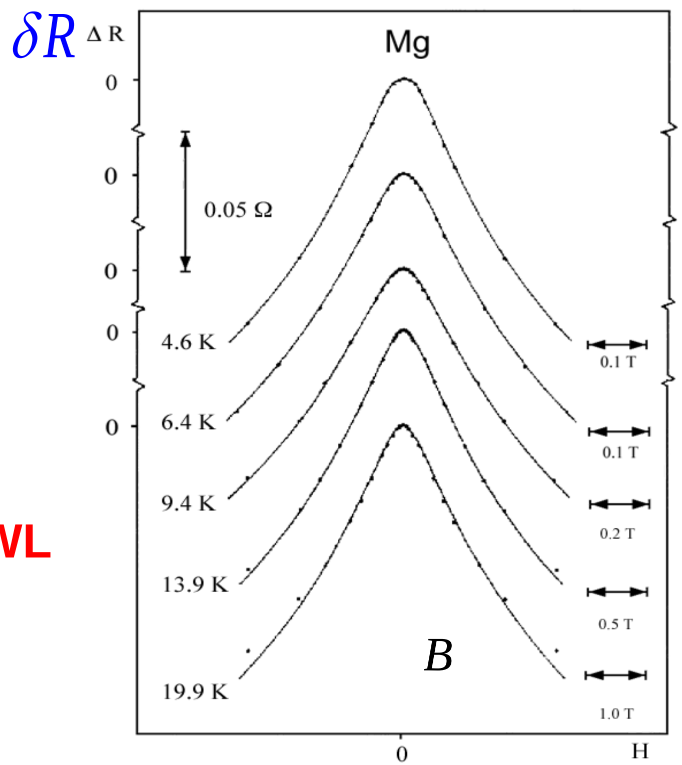
$$\delta G = \frac{e^2}{\pi h}\left[\ln\left(\frac{B}{B_\phi}\right) + \psi\left(\frac{1}{2}+\frac{B_\phi}{B}\right)\right] = \frac{e^2}{\pi h}F\left(\frac{B}{B_\phi}\right)$$

No spin-orbit coupling

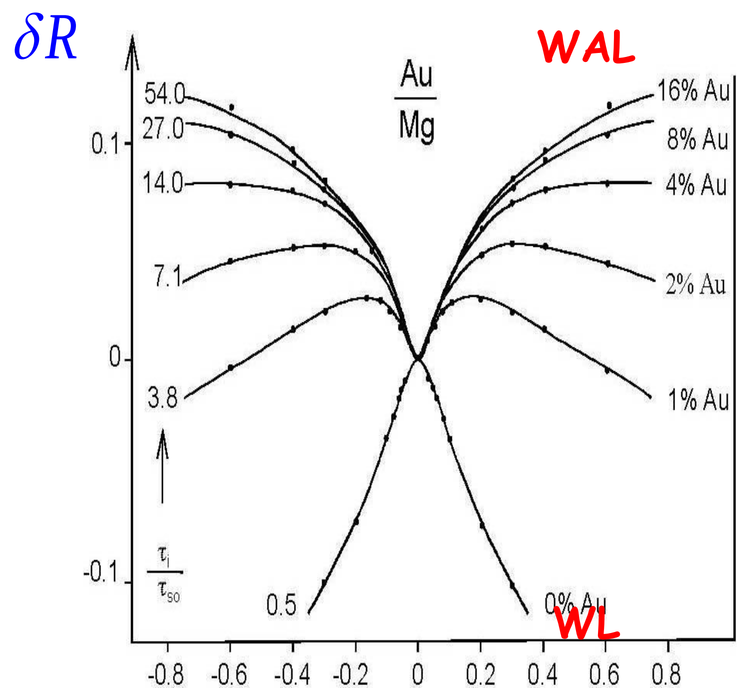
Spin-orbit coupling

Bergmann, 84

WL



Bergmann, 84



Positive magnetoconductance

weak-localization

$$\delta\sigma = \frac{e^2}{\pi h} F\left(\frac{B}{B_\phi}\right)$$

Negative magnetoconductance

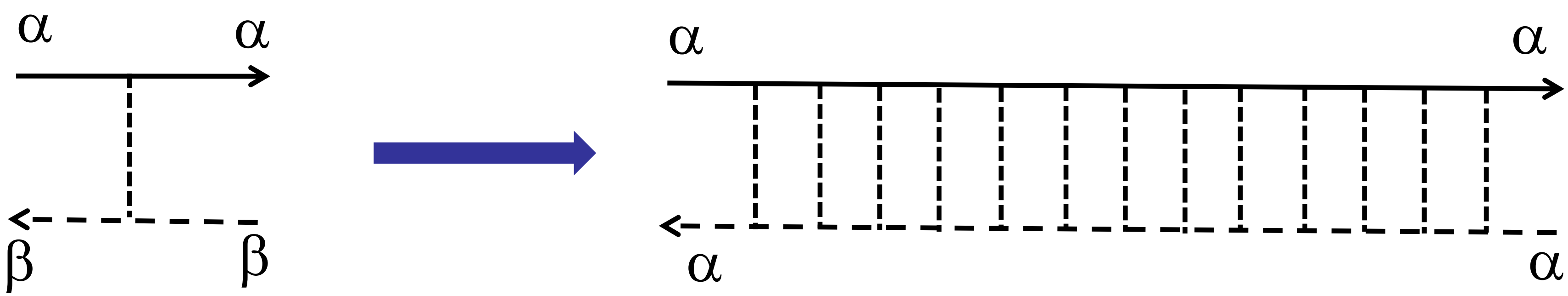
weak-antilocalization

$$\delta\sigma = \frac{e^2}{2\pi h}\left[3F\left(\frac{B}{B_\phi + B_{so}}\right) - F\left(\frac{B}{B_\phi}\right)\right]$$

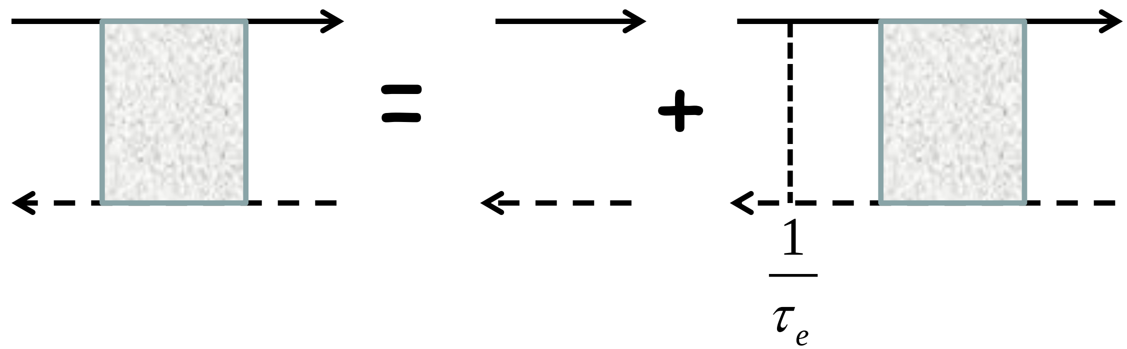
Triplet Singlet

Hikami, Larkin and Nagaoka,
Prog. Theor. Phys. 63,707 (1980)

Structure itérative du Cooperon : absence de spin-orbite



$$P = P_0 + P_0 \frac{1}{\tau_e} P$$



$$P = \frac{P_0}{1 - P_0 / \tau_e}$$

$$P_0 = \tau_e (1 + i\omega\tau_e - Dq^2\tau_e)$$

$$P(q,\omega) = \frac{1}{-i\omega + Dq^2}$$

Couplage spin-orbite

Nouveau temps caractéristique

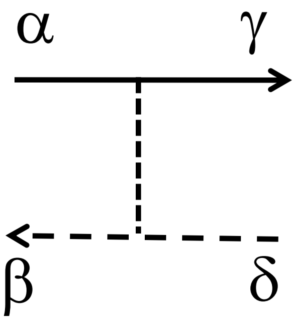
$$\tau_{so}$$

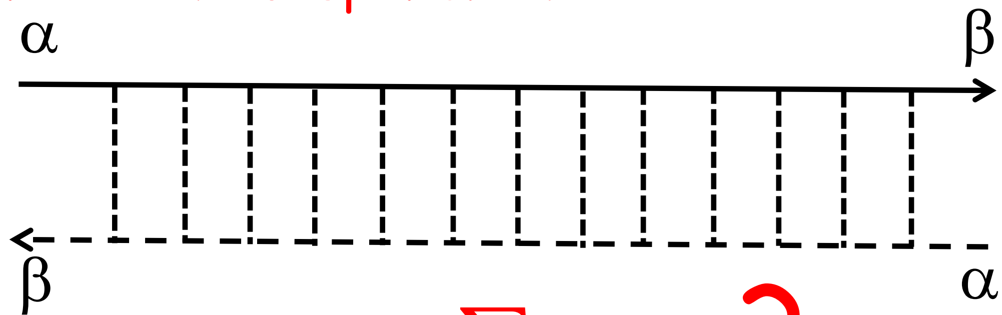
Temps moyen sur lequel un électron perd la direction de son spin initial

$$\frac{1}{\tau'_e} = \frac{1}{\tau_e} + \frac{1}{\tau_{so}}$$

$$P'_0 = \tau'_e (1 + i\omega\tau'_e - Dq^2\tau'_e)$$

Structure itérative du Cooperon : avec spin-orbite

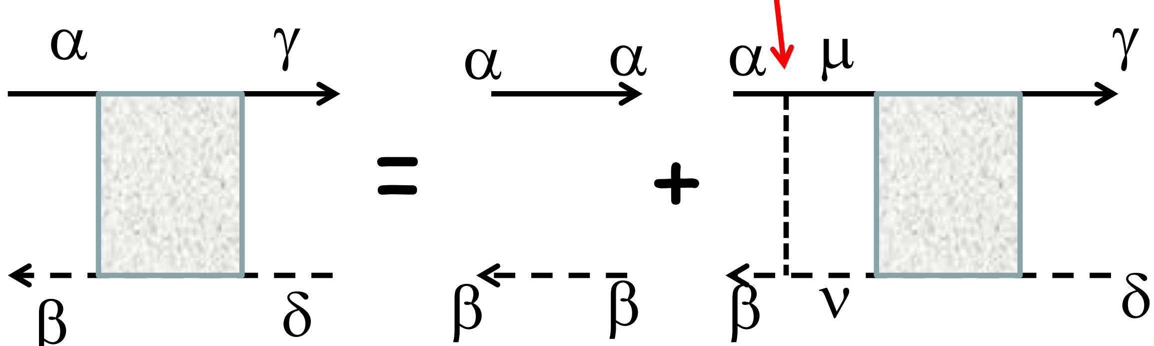




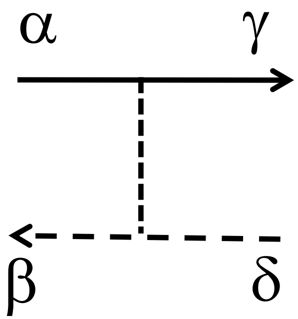
$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha} \quad ?$$

$$\frac{1}{\tau_e} \longrightarrow \frac{b_{\alpha\beta,\gamma\delta}}{\tau_e} = \frac{\delta_{\alpha\gamma}\delta_{\beta\delta}}{\tau_e} - \frac{1}{3\tau_{so}} \vec{\sigma}_{\alpha\gamma} \cdot \vec{\sigma}_{\beta\delta}$$

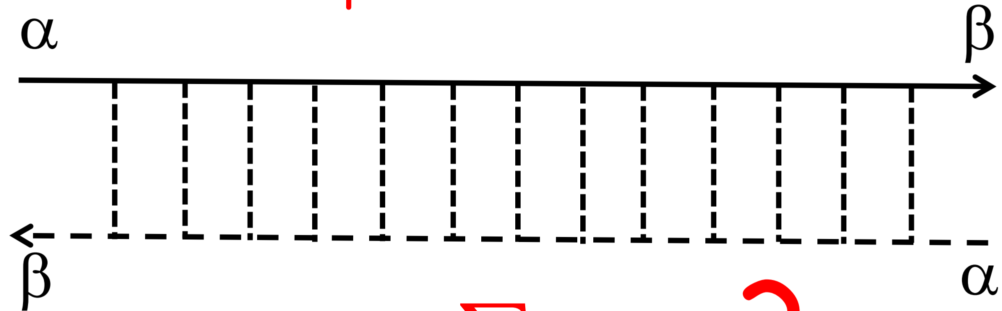
$$P_{\alpha\beta,\gamma\delta} = P_0^{'} \delta_{\alpha\gamma}\delta_{\beta\delta} + \sum_{\mu\nu} P_0^{'} \frac{b_{\alpha\beta,\mu\nu}}{\tau_e} P_{\mu\nu,\gamma\delta}$$



Structure itérative du Cooperon : avec spin-orbite



?



$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha} \quad ?$$

$$\frac{1}{\tau_e} \longrightarrow \frac{b_{\alpha\beta,\gamma\delta}}{\tau_e} = \frac{\delta_{\alpha\gamma}\delta_{\beta\delta}}{\tau_e} - \frac{1}{3\tau_{so}} \vec{\sigma}_{\alpha\gamma} \cdot \vec{\sigma}_{\beta\delta}$$

$$P_{\alpha\beta,\gamma\delta} = P_0^{'} \delta_{\alpha\gamma}\delta_{\beta\delta} + \sum_{\mu\nu} P_0^{'} \frac{b_{\alpha\beta,\mu\nu}}{\tau_e} P_{\mu\nu,\gamma\delta}$$

Diagonaliser…

$$b_{\alpha\beta,\gamma\delta} = \sum_{Jm} b_{Jm} \langle \alpha\beta | Jm \rangle \langle Jm | \gamma\delta \rangle$$

J=0 singulet
J=1 triplet

$$P_{Jm} = P_0^{'} + P_0^{'} \frac{b_{Jm}}{\tau_e} P_{Jm} \longrightarrow P_J = \frac{P_0^{'}}{1 - b_J P_0^{'} / \tau_e}$$

$$P_J = \frac{P_0^{'}}{1 - b_J P_0^{'} / \tau_e} \qquad P_0^{'} = \tau_e^{'}(1 + i\omega\tau_e^{'} - Dq^2\tau_e^{'})$$

$$P_J = \frac{1}{1/\tau_J - i\omega + Dq^2}$$

avec

$$\frac{1}{\tau_J} = \frac{1}{\tau_e}\left(1 - b_J \frac{\tau_e^{'}}{\tau_e}\right)$$

Reste à déterminer

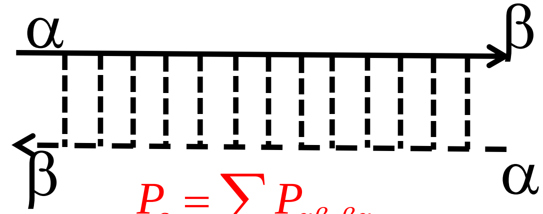
$$b_{Jm} = \sum_{\alpha\beta,\gamma\delta} b_{\alpha\beta,\gamma\delta} \langle Jm|\alpha\beta\rangle\langle\gamma\delta|Jm\rangle$$

$$\frac{b_{\alpha\beta,\gamma\delta}}{\tau_e} = \frac{\delta_{\alpha\gamma}\delta_{\beta\delta}}{\tau_e} - \frac{1}{3\tau_{so}} \vec{\sigma}_{\alpha\gamma} \cdot \vec{\sigma}_{\beta\delta} \Rightarrow \frac{b_S}{\tau_e} = \frac{1}{\tau_e} + \frac{1}{\tau_{so}} \qquad \frac{b_T}{\tau_e} = \frac{1}{\tau_e} - \frac{1}{3\tau_{so}}$$

Temps caractéristiques :

$$\frac{1}{\tau_S} = 0 \qquad \frac{1}{\tau_T} = \frac{4}{3\tau_{so}}$$

$$P_{\alpha\beta,\gamma\delta} = \sum_{Jm} P_{Jm} \langle \alpha\beta | Jm \rangle \langle Jm | \gamma\delta \rangle$$



$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha}$$

$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha} = \sum_{\beta} \sum_{Jm} P_{Jm} \langle \alpha\beta | Jm \rangle \langle Jm | \beta\alpha \rangle$$

$$P_c = \sum_{\beta} P_{\uparrow\beta,\beta\uparrow} = \sum_{Jm} P_{Jm} \Big(\langle \uparrow\uparrow | Jm \rangle \langle Jm | \uparrow\uparrow \rangle + \langle \uparrow\downarrow | Jm \rangle \langle Jm | \downarrow\uparrow \rangle \Big)$$

$$|1,1\rangle = |\uparrow\uparrow\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}} \big(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \big)$$

$$|1,-1\rangle = |\downarrow\downarrow\rangle$$

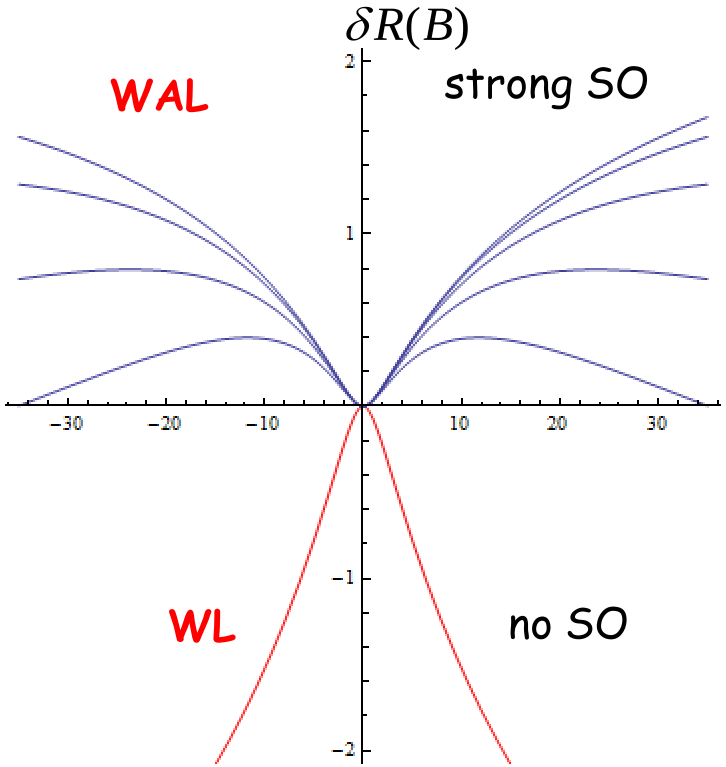
$$|0,0\rangle = \frac{1}{\sqrt{2}} \big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \big)$$

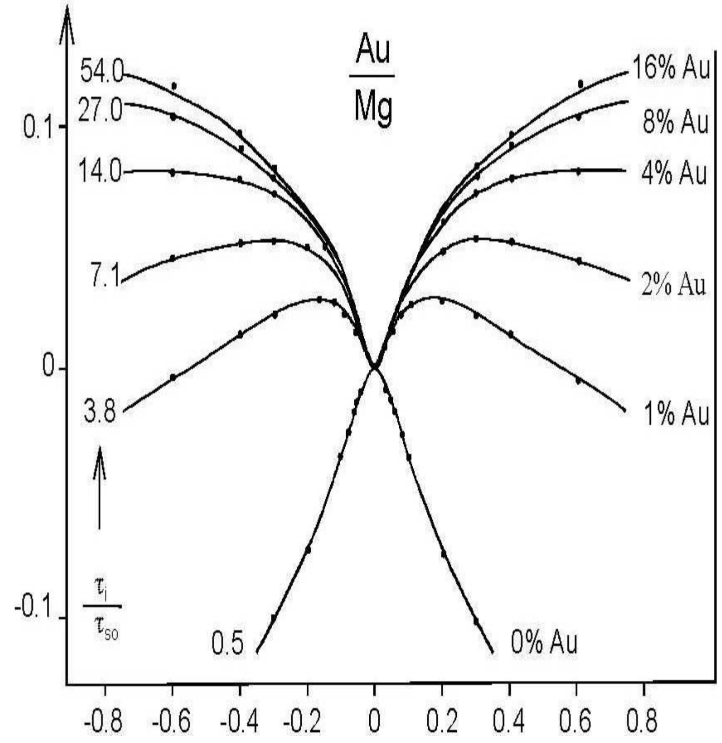
$$P_c(q,\omega) = \frac{3}{2} P_T(q,\omega) - \frac{1}{2} P_S(q,\omega)$$

Triplet Singulet

$$P_c(q,t) = \frac{1}{2}\left(3e^{-t/\tau_T} - 1\right) e^{-Dq^2 t}$$

$$\frac{1}{\tau_T} = \frac{4}{3\tau_{so}}$$

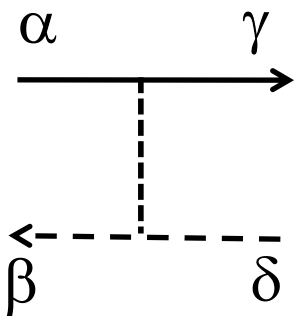




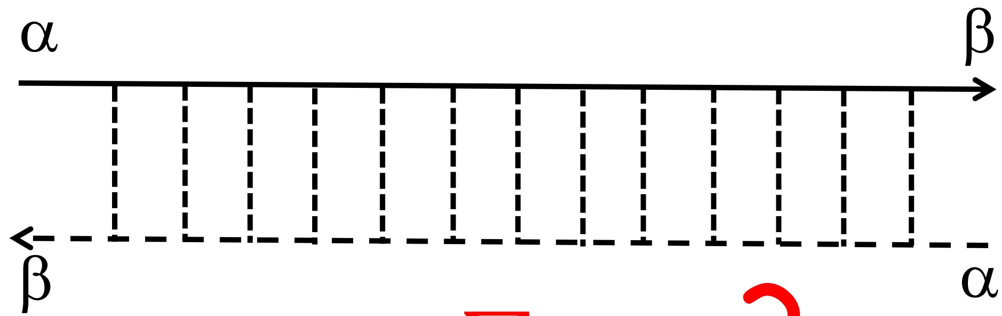
$$\delta\sigma = \frac{e^2}{2\pi h}\left[3F\left(\frac{B}{B_\phi + B_{so}}\right) - F\left(\frac{B}{B_\phi}\right)\right]$$

Hikami et al., Prog. Theor. Phys. 63,707 (1980)

Impuretés magnétiques



?



$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha} \quad ?$$

$$\frac{1}{\tau_e} \Rightarrow \frac{b_{\alpha\beta,\gamma\delta}}{\tau_e} = \frac{\delta_{\alpha\gamma}\delta_{\beta\delta}}{\tau_e} + \frac{1}{3\tau_m}\vec{\sigma}_{\alpha\gamma}\cdot\vec{\sigma}_{\beta\delta}$$

$$P_{\alpha\beta,\gamma\delta} = P_0^{'}\,\delta_{\alpha\gamma}\delta_{\beta\delta} + \sum_{\mu\nu} P_0^{'}\,\frac{b_{\alpha\beta,\mu\nu}}{\tau_e} P_{\mu\nu,\gamma\delta}$$

Diagonaliser…

$$b_{\alpha\beta,\gamma\delta} = \sum_{Jm} b_{Jm} \langle \alpha\beta | Jm \rangle \langle Jm | \gamma\delta \rangle$$

J=0 singulet
J=1 triplet

$$P_{Jm} = P_0^{'} + P_0^{'}\,\frac{b_{Jm}}{\tau_e} P_{Jm} \Rightarrow P_J = \frac{P_0^{'}}{1 - b_J P_0^{'} / \tau_e}$$

$$P_J = \frac{P_0'}{1 - b_J P_0' / \tau_e}$$

$$P_0' = \tau_e'(1 + i\omega\tau_e' - Dq^2\tau_e')$$

$$\boxed{P_J = \frac{1}{1/\tau_J - i\omega + Dq^2}}$$

avec

$$\frac{1}{\tau_J} = \frac{1}{\tau_e}\left(1 - b_J \frac{\tau_e'}{\tau_e}\right)$$

Reste à déterminer

$$b_{Jm} = \sum_{\alpha\beta,\gamma\delta} b_{\alpha\beta,\gamma\delta} \langle Jm|\alpha\beta\rangle\langle\gamma\delta|Jm\rangle$$

$$\frac{b_{\alpha\beta,\gamma\delta}}{\tau_e} = \frac{\delta_{\alpha\gamma}\delta_{\beta\delta}}{\tau_e} + \frac{1}{3\tau_m}\vec{\sigma}_{\alpha\gamma} \cdot \vec{\sigma}_{\beta\delta}$$

$$\frac{b_S}{\tau_e} = \frac{1}{\tau_e} - \frac{1}{\tau_m}$$

$$\frac{b_T}{\tau_e} = \frac{1}{\tau_e} + \frac{1}{3\tau_m}$$

Temps caractéristiques :

$$\boxed{\frac{1}{\tau_S} = \frac{2}{\tau_m}}$$

$$\boxed{\frac{1}{\tau_T} = \frac{2}{3\tau_m}}$$

$$P_{\alpha\beta,\gamma\delta} = \sum_{Jm} P_{Jm} \langle \alpha\beta | Jm \rangle \langle Jm | \gamma\delta \rangle$$

$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha} = \sum_{\beta} \sum_{Jm} P_{Jm} \langle \alpha\beta | Jm \rangle \langle Jm | \beta\alpha \rangle$$

$$P_c = \sum_{\beta} P_{\alpha\beta,\beta\alpha} = \sum_{Jm} P_{Jm} \Big(\langle \uparrow\uparrow | Jm \rangle \langle Jm | \uparrow\uparrow \rangle + \langle \uparrow\downarrow | Jm \rangle \langle Jm | \downarrow\uparrow \rangle \Big)$$

$$|1,1\rangle = |\uparrow\uparrow\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}} \big(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \big)$$

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$$|0,0\rangle = \frac{1}{\sqrt{2}} \big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \big)$$

$$P_c(q,\omega) = \frac{3}{2} P_T(q,\omega) - \frac{1}{2} P_S(q,\omega)$$

Triplet Singulet

$$P_c(q,t) = \frac{1}{2} \left(3e^{-t/\tau_T} - e^{-t/\tau_S} \right) e^{-Dq^2 t}$$

Magnétorésistance d'un plan 2D avec spin-orbite et impuretés magnétiques

$$\delta\sigma = \frac{e^2}{2\pi h}\left[3F\left(\frac{B}{B_\phi + B_{so} + B_{m,T}}\right) - F\left(\frac{B}{B_\phi + B_{m,S}}\right)\right]$$

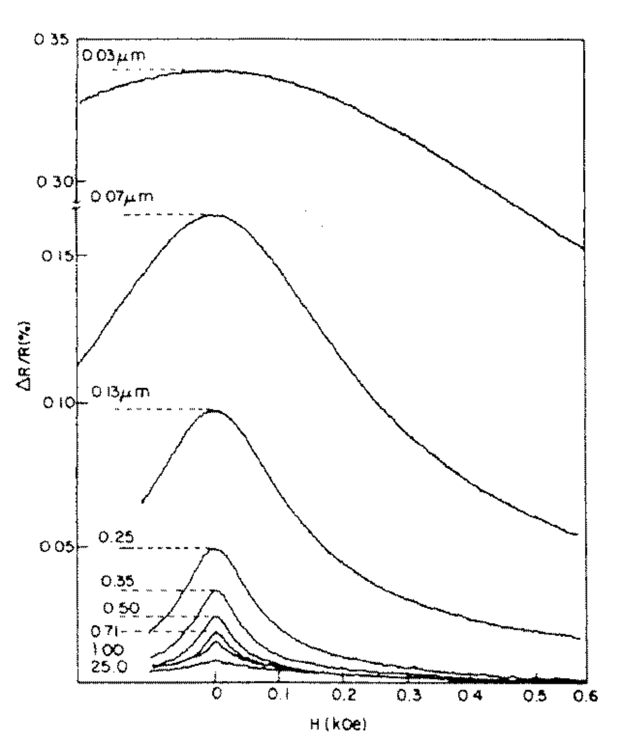
$$F(x) = \ln x + \psi\left(\frac{1}{2} + \frac{1}{x}\right)$$

Dephasing by e-e interactions

Temperature dependence of the phase coherence length

$$L_\phi(T) \quad ??$$

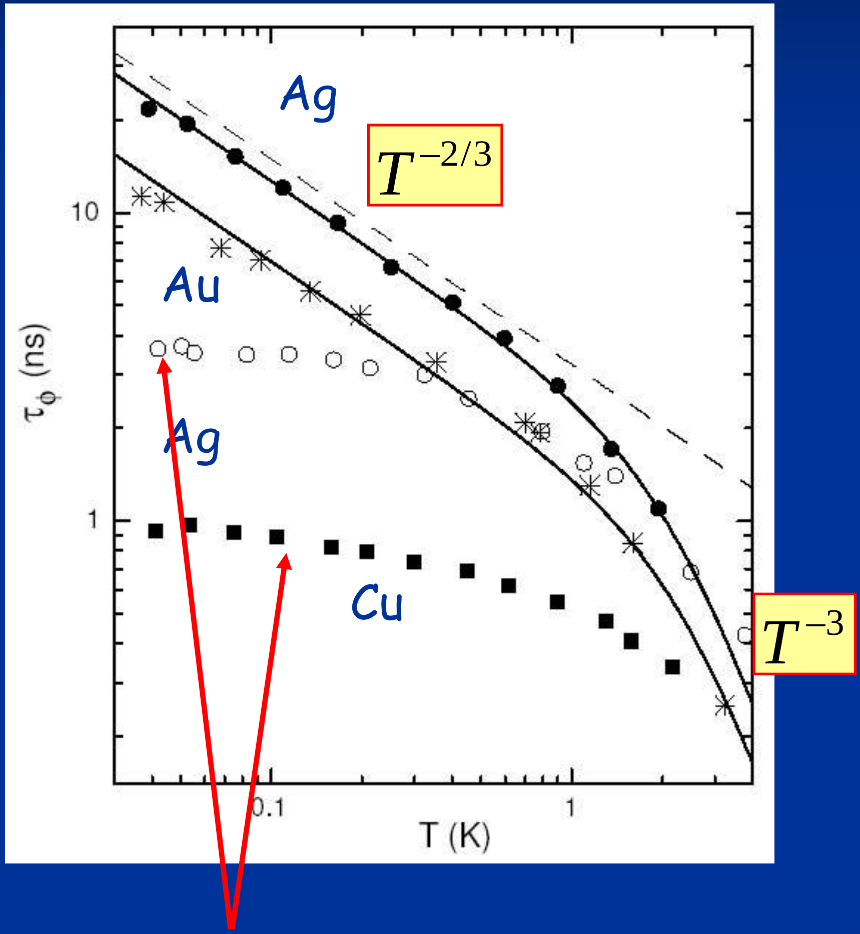
W.L. in a quasi-1D wire



Licini,Dolan,Bishop,1980

$$B^{*}W\,L_\phi(T) \sim \phi_0 \quad \longrightarrow \quad L_\phi \propto T^{-1/3}$$

Déphasage dans des fils métalliques



e-e e-phonon

$$\frac{1}{\tau_\phi(T)} = A T^{2/3} + B T^3$$

Saturation due aux impuretés magnétiques

Saclay

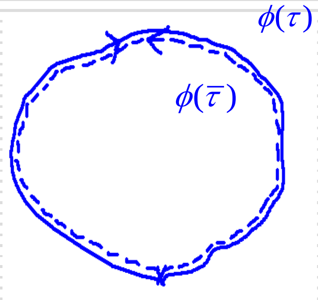
Dephasing by e-e interactions

B.L. Altshuler, A.G. Aronov, D.E. Khmelnitskii, J. Phys. *C* **15**, 7367 (1982)

Effects of electron-electron collisions with small energy transfers on quantum localization

Dephasing by e-e interactions

Weak-localization correction :



$$\Delta g = -4\int P_0(t)\ e^{-t/\tau_\phi}\ \frac{dt}{\tau_D}$$

$$\Delta g = -4\int P_0(t)\ \left\langle e^{i\Phi(t)} \right\rangle \frac{dt}{\tau_D}$$

Dephasing : e-e interaction

quasi-1D wire

Phase coherence time $\tau_\phi \propto T^{-2/3}$

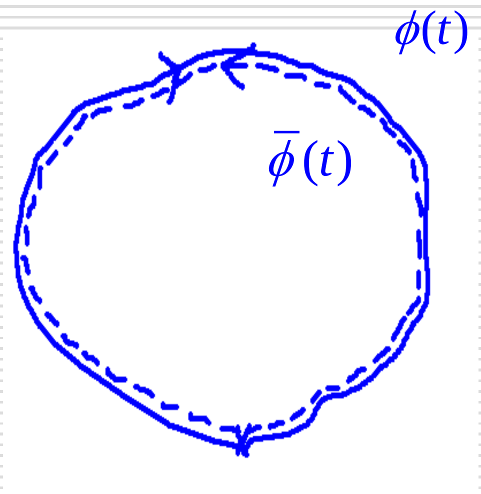
Altshuler,Aronov,Khmelnitskii

AAK qualitatif

quasi-1d wire

$$\left\langle e^{i\Phi(t)} \right\rangle \quad ?$$

e-e interaction = electric fluctuating potential → Fluctuating phase



$$\left\langle e^{i\Phi(t)} \right\rangle \sim e^{-\frac{1}{2}\left\langle \Phi^2(t) \right\rangle}$$

$$\Phi(t) = \phi(t) - \bar{\phi}(t)$$

$$\phi(t) = \frac{e}{\hbar}\int_0^t V(r(\tau),\tau)\, d\tau$$

$$\left\langle \Phi^2(t) \right\rangle = 2\frac{e^2}{\hbar^2}\int_0^t \left[\left\langle V(r_\tau,\tau)V(r_\tau,\tau) \right\rangle - \left\langle V(r_\tau,\tau)V(r_\tau,\bar{\tau}) \right\rangle \right] d\tau$$

$$\frac{d\left\langle \Phi^2(t) \right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\left\langle V^2 \right\rangle_t$$

AAK qualitatif

Nyquist theorem

$$\frac{d\left\langle \Phi^2(t)\right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\left\langle V^2\right\rangle_t$$

$$\left\langle V^2\right\rangle_t \sim k_B T\, R_t \sim k_B T\,\frac{r_t}{\sigma_0 S}$$

$$\frac{d\left\langle \Phi^2(t)\right\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0 S}\, r_t$$

Diffusion

$$r_t \sim \sqrt{Dt}$$

$$\left\langle \Phi^2(t)\right\rangle \sim \frac{k_B T\sqrt{D}}{\sigma_0 S}\, t^{3/2} \sim \left(\frac{t}{\tau_N}\right)^{3/2}$$

$$\tau_N \sim \left(\frac{\hbar^2\sigma_0 S}{e^2 k_B T\sqrt{D}}\right)^{2/3} \propto \frac{1}{T^{2/3}}$$

Nyquist time (Aronov, Altshuler, Khmelnitskii)

$$\left\langle e^{i\Phi(t)}\right\rangle \sim e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle} \sim e^{-(t/\tau_N)^{3/2}}$$

AAK résultat exact

approché

Fluctuations thermiques trajectoires

$$\left\langle \left\langle e^{i\Phi(t)} \right\rangle_T \right\rangle_C = \left\langle e^{-\frac{1}{2}\left\langle \Phi^2(t)\right\rangle_T} \right\rangle_C$$

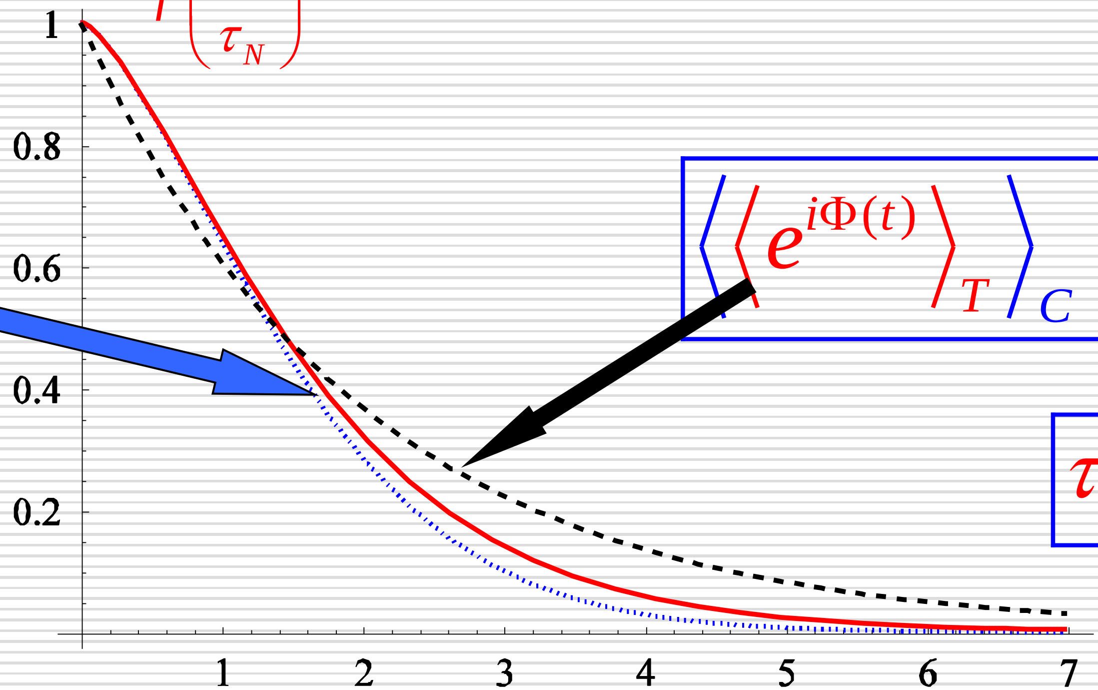
$$e^{-\frac{1}{2}\left\langle\left\langle \Phi^2(t)\right\rangle_T\right\rangle_C}$$

$$= e^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2}}$$

$$f\left(\frac{t}{\tau_N}\right)$$

$$e^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2}}$$

$$\left\langle \left\langle e^{i\Phi(t)} \right\rangle_T \right\rangle_C \sim e^{-t/2\tau_N}$$



$$\tau_\phi = 2\tau_N$$

G.M., E. Akkermans, *Phys. Rev. Lett.* **95**, 016403 (2005)

AAK qualitatif dimensions 1, 2, 3

$$\frac{d\langle \Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\langle V^2\rangle_t$$

$$\langle V^2\rangle_t \sim k_B T\, R_t \sim k_B T \frac{r_t}{\sigma_0 S}$$

d=1

d=2

d=3

$$\frac{d\langle \Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0 S}\sqrt{Dt}$$

$$\frac{d\langle \Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0}$$

$$\frac{d\langle \Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0 \sqrt{Dt}}$$

Coupure thermique aux temps courts

$$\frac{d\langle \Phi^2(t)\rangle}{dt} \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0}\sqrt{\frac{T}{\hbar D}}$$

$$\langle \Phi^2(t)\rangle \sim \frac{k_B T\sqrt{D}}{\sigma_0 S}\, t^{3/2} \sim \left(\frac{t}{\tau_N}\right)^{3/2}$$

$$\langle \Phi^2(t)\rangle \sim \frac{k_B T}{\sigma_0}\, t \sim \frac{t}{\tau_N}$$

$$\langle \Phi^2(t)\rangle \sim \frac{e^2}{\hbar^2}\frac{k_B T}{\sigma_0}\sqrt{\frac{T}{\hbar D}}\, t$$

$$\tau_N \propto \frac{1}{T^{2/3}}$$

$$\tau_N \propto \frac{1}{T}$$

$$\tau_N \propto \frac{1}{T^{3/2}}$$