Direct Measurement of the Phase-Coherence Length in a GaAs/GaAlAs Square Network

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The low temperature magnetoconductance of a large array of quantum coherent loops exhibits Altshuler-Aronov-Spivak oscillations with a periodicity corresponding to 1/2 flux quantum per loop. We show that the measurement of the harmonics content provides an accurate way to determine the electron phase-coherence length L_φ in units of the lattice length with no adjustable parameters. We use this method to determine L_φ in a square network realized from a 2D electron gas in a GaAs/GaAlAs heterojunction, with only a few conducting channels. The temperature dependence follows a power law $T^{-1/3}$ from 1.3 K to 25 mK with no saturation, as expected for 1D diffusive electronic motion and electron-electron scattering as the main decoherence mechanism.

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The characteristic scale on which quantum interference can occur in a conductor, the phase-coherence length L_φ, is the key parameter of quantum transport. In particular, the dependence of L_φ on temperature can discriminate between the various scattering mechanisms which limit phase coherence: electron-electron (*e-e*), electron-phonon, or electron-magnetic impurity interactions. Interference on the scale of L_φ gives rise to two different types of contributions to the conductance in a transport experiment. Some are sample specific and depend on the particular disorder configuration. These are conductance fluctuations (magnetofingerprints) and the ϕ_0 periodic Aharonov-Bohm (AB) oscillations ($\phi_0 = h/e$ is the flux quantum). Their amplitudes are governed both by L_φ and the thermal length L_T, in general smaller than L_φ. This makes an accurate determination of L_φ difficult [1–3]. The second type of contribution, called the weak localization (WL) correction, is obtained after ensemble averaging of quantum interferences on many configurations of disorder. It originates from interferences between time reversed electronic trajectories, which are the only ones surviving the disorder average. It is also observed in samples of size $L \gg (L_\varphi, L_T)$ and depends only on L_φ since it involves trajectories at the same energy. Manifestations of WL are the magnetoconductance (MC) of large connex samples [1–3] and the Altshuler-Aronov-Spivak (AAS) $\phi_0/2$ periodic oscillations resulting from the ensemble average of AB oscillations in a long cylinder or large arrays of connected phase coherent rings [4–6]. The WL provides thus in general a much more direct measurement of L_φ than sample specific corrections.

The analysis of the MC in 1D diffusive metallic wires (with transverse dimensions smaller than L_φ) has led to accurate determinations of L_φ. It was found that the dominant phase breaking mechanism at very low temperature, in the absence of magnetic impurities, is due to *e-e* scattering and is well described by the Altshuler-Aronov-Khmelnitsky (AAK) theory [1,7] yielding $L_\varphi \propto T^{-1/3}$ with no saturation down to 40 mK [2,3]. Such a remarkable agreement between theory and experiment has not been established for semiconducting wires, where most WL experiments have been performed only above 0.2 K or with insufficient ensemble averaging [8–11]. It is, however, essential to check the validity of the AAK theory for these systems which correspond to radically different physical parameters: fewer conducting channels and larger screening lengths. In this Letter we present MC data down to 25 mK of networks fabricated from a GaAs/GaAlAs 2D electron gas (2DEG), which contain 10^6 square loops in the diffusive transport regime, and determine L_φ *without adjustable parameters* from the analysis of the AAS oscillations (Fig. 1). Following [12,13], we explain how to calculate the harmonics content of these oscillations and show that it depends only on L/L_φ where L is the circumference of the elementary loop. It is then possible to determine L_φ and its temperature dependence exclusively from geometrical parameters of the network. This new method of determinating L_φ is especially interesting in these 2DEG wires for which basic transport parameters such as the electron density and wire width (W) are not straightforwardly determined, unlike metals.

Moreover, once L_φ is determined, we deduce from the analysis of the high field positive MC the elastic mean free path (l_e), W, and make a detailed comparison with theoretical predictions of the AAK theory on dephasing by *e-e* interactions. We find a very good quantitative agreement in the regime, never explored before, of very few conducting channels.

In the weakly disordered diffusive regime ($k_F l_e \gg 1$), the WL correction is directly related to the Cooperon, which can be computed from the time integrated return probability $P_c(\vec{r}, \vec{r})$ for a diffusive electron. In a cylinder

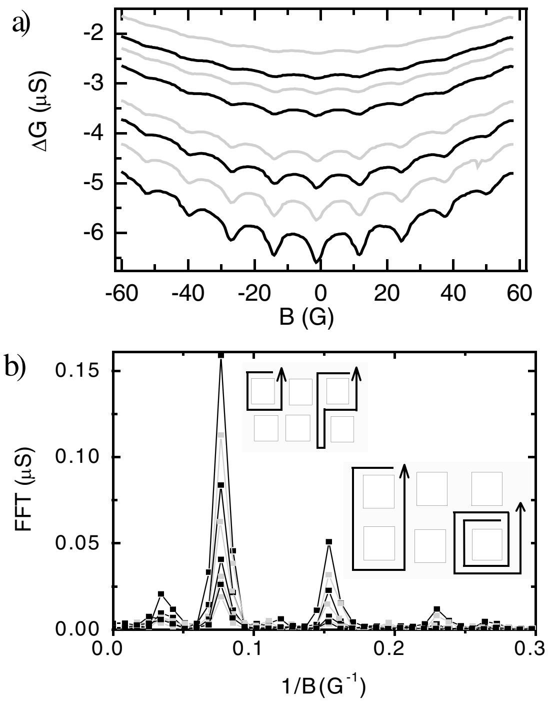

FIG. 1. (a) Conductance versus magnetic field between 25 and 220 mK. (b) Fourier transform of the MC (after substraction of the envelope) for different temperatures. Left (right) inset: Some orbits contributing to the first (second) harmonic.

or an array of connected loops the contribution to the Cooperon of trajectories enlacing at least one loop oscillates with a flux periodicity of $\phi_0/2$ giving rise to the AAS oscillations. A systematic way of calculating WL in a mesoscopic network of diffusive wires was derived in [12]. More recently [13], a relation was found between the conductivity WL correction (divided by e^2/h) and the spectral determinant $S(\gamma) = \det(\gamma - \Delta)$ of the Laplace operator Δ defined on the network.

$$\Delta\sigma = -4\int \frac{d\vec{r}}{\mathrm{Vol}} P_c(\vec{r}, \vec{r}) = \frac{-4}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma), \qquad (1)$$

where Vol is the total volume and $\gamma = 1/L_\varphi^2$. Equation (1) assumes an exponential relaxation of phase coherence. This approach, which is meaningful only for regular networks, is particularly efficient because $S(\gamma)$ can be computed systematically for any given network in terms of the determinant of a finite size matrix encoding the network's characteristics (topology, length of the wires, magnetic flux). It can also be shown that the WL can be expressed, in the small L_φ limit, as a trace expansion over periodic orbits, C,

$$\frac{\partial \ln S(\gamma)}{\partial\gamma} = \frac{1}{2\sqrt{\gamma}}\left[\mathcal{L} + \frac{V-B}{\sqrt{\gamma}} + \sum_C l(\tilde{C})\alpha(C)e^{-\sqrt{\gamma}l(C)+i\theta(C)}\right],$$

where V (B), is the total number of vertices (bonds). $\tilde{C}$ is the primitive orbit related to C. We explain briefly this formula, demonstrated in [14] and discussed in detail in [15]. Each orbit contributes to the MC with a phase factor which depends on the enclosed flux: $\theta(C) = 4\pi\Phi(C)/\phi_0$. It is also characterized by its length $l(C)$ and by a geometrical weight $\alpha(C)$. In the case of a square lattice of periodicity a, the periodically oscillating conductance can be decomposed in Fourier space as a sum of harmonics of the fundamental periodicity corresponding to $\phi_0/2$ per elementary cell. The first terms of this expansion in $x = e^{-2a/L_\varphi}$ read

$$\Delta\sigma = \frac{-L_\varphi}{W}\left[2 - \frac{L_\varphi}{a} + x + \cdots + \frac{x^2}{2}\cos\theta\left(1 - \frac{3}{2}x + \ldots\right) + \frac{3x^3}{8}\cos 2\theta\left(1 - \frac{19}{12}x + \ldots\right) + \frac{3x^4}{8}\cos 3\theta\left(1 - \frac{15}{8}x + \ldots\right)\right]. \qquad (2)$$

Here W is the section of the wire, $\theta = 4\pi\phi/\phi_0$, and ϕ is the flux per elementary cell. The amplitude of the nth harmonics is evaluated by counting the paths enclosing n fluxes ϕ. The counting is rapidly cumbersome (156 orbits are involved in the last term), but the crucial point is that the coefficient of each term depends only on the lattice geometry.

More generally, the WL correction can be obtained for all values of L_φ/a from the numerical computation of the determinant in Eq. (1). The numerical fast Fourier transform of the computed MC yields the ratio R_{12} of the two first harmonics as a function of L_φ/a (Fig. 2, inset). It appears that the small orbit expansion (2) is a good approximation up to $L_\varphi/a = 2$. In any case the ratio of two harmonics is completely determined by L_φ/a and provides a method for a direct evaluation of L_φ without any adjustable parameter. The square lattice is particularly appropriate for such a determination of L_φ due to its large harmonics content: the second harmonic is dominated by orbits of length $6a$ instead of $8a$ for a statistical ensemble of single rings or a necklace of identical rings, for example.

We now use this method to determine the phase-coherence length of square networks etched in a 2DEG of a GaAs/GaAlAs heterostructure. The networks consist of 10^6 square loops of side $a = 1$ μm and nominal width $W_0 = 0.5$ μm and cover a total area of 1 mm^2. A gold gate deposited 100 nm above the 2DEG offers the possibility to change the number of electrons in the network. Measurements were done on three networks (A, B with gate, C without), giving the same results. Except when specified, figures show the data for sample A. We have measured the MC up to 4.5 T between 25 mK and 1.3 K, using a standard lock-in technique (ac current of 1 nA at 30 Hz). The samples were, in general, strongly depleted at low temperature because of the etching. The intrinsic electron density of the 2DEG, $n_e = 4.4 \times 10^{15}$ m^{-2}, was recovered after illuminating the samples during

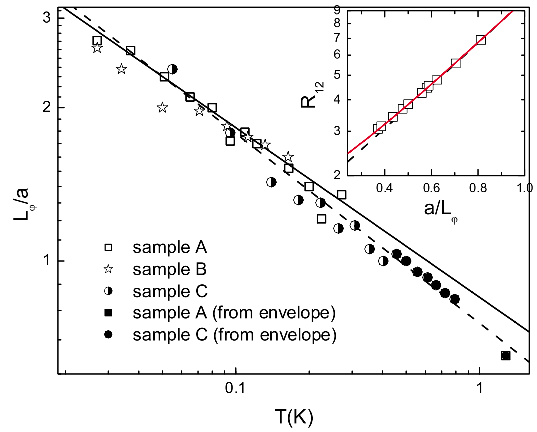

FIG. 2 (color online). Inset: Relation between L_φ/a and the ratio R_{12} of the two first harmonics on a semilog scale. The continuous line comes from the numerical calculation of $S(\gamma)$. The dashed line is deduced from the expansion (2). The open circles are the experimental values of R_{12} from which L_φ is determined. Main panel: L_φ versus temperature on a log-log scale obtained from R_{12} for samples A, B, and C. The fit (dashed line) yields the power law $L_\varphi \propto T^{-0.36}$. The dark circles and squares are obtained from the fit of the envelope. The continuous line is L_φ from the AAK theory (for sample A).

several minutes at 4.2 K. This density was determined from Shubnikov–de Haas oscillations visible above 1 T. Because of depletion after etching of the 2DEG, it is difficult to estimate the real width of the wires (W) and l_e.

At low magnetic field [Fig. 1(a)], the MC exhibit large AAS oscillations with a period 12.6 G corresponding to a flux $\phi_0/2$ in a square cell of area a^2. The oscillations are clearly not purely sinusoidal. At the lowest temperature, 25 mK, three harmonics are visible in the Fourier spectrum of the MC [Fig. 1(b)]. Moreover, as shown in Fig. 3 which represents the MC for a wider range of field, the oscillations disappear above 60 G but the WL magnetoconductance due to the penetration of the field through the finite width of the wires constituting the network is still clearly visible. At high temperature, above 400 mK, the AAS oscillations disappear even at low field. Only the positive MC remains with a smaller amplitude. In sample B, the same experiments for different gate voltages were also performed.

We first concentrate on the AAS oscillations [Fig. 1(a)]. The Fourier spectrum of the MC exhibits a series of peaks corresponding to successive harmonics of the $\phi_0/2$ periodicity. The finite width of the peaks [Fig. 1(b)] is due to the penetration of the magnetic field in the wires which damps the AAS oscillations at high field. It can be shown that this broadening does not affect the integral of the peak. A first rough analysis shows that the ratio R_{12} of integrated peaks of the two first harmonics behaves like $R_{12} \sim \exp(T^{1/3})$. We now use the theory described above to quantitatively determine L_φ via the relation between L_φ/a and R_{12}. We deduce its temperature dependence between 25 and 250 mK as shown in

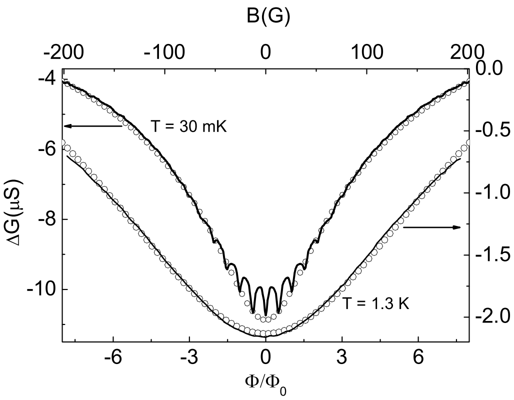

FIG. 3. High field MC, the continuous lines are the experimental data, and the dots are the fits with Eq. (4). Parameters of the fits are l_e and W at 30 mK, and L_φ and l_e at 1.3 K.

Fig. 2. We find that L_φ follows a power law $T^{-\eta}$, where $\eta = 0.36 \pm 0.05$. The coherence length reaches almost 3 μm at 25 mK, and there is no sign of saturation.

Once L_φ is determined, the sample parameters (W, l_e) can be deduced from the WL envelope. The magnetic field appears as an additional effective phase breaking rate for the time reversed trajectories responsible for the WL leading to an effective L_φ given by [16]

$$\frac{1}{L_\varphi(\phi)^2} = \frac{1}{L_\varphi(0)^2}\left[1 + \frac{1}{3}\left(2\pi\frac{\phi}{\phi_0}\frac{W_{\mathrm{eff}}L_\varphi(0)}{a^2}\right)^2\right], \quad (3)$$

where $W_{\mathrm{eff}} = W\sqrt{(3W)/(C_1 l_e)}$ is a renormalized width which appears in the WL correction for a semiballistic wire ($l_e \gg W$) due to the phenomenon of flux cancellation. The coefficient C_1 depends on the specific boundary conditions. The samples under consideration are close to the case of specular boundary scattering [8] for which $C_1 = 9.5$. The MC envelope, given by $\langle\Delta\sigma(\phi = 0, L_\varphi(\phi))\rangle$, can be analytically computed for the square lattice geometry and is given by

$$\Delta\sigma = \frac{-L_\varphi}{W}\left[\coth\frac{a}{L_\varphi} - \frac{L_\varphi}{a} + \frac{2}{\pi}\tanh\frac{a}{L_\varphi}K\left(\frac{1}{\cosh\frac{a}{L_\varphi}}\right)\right], \quad (4)$$

where $K(x)$ is a complete elliptic integral. This expression is used in a two-parameter (W, σ_{D}) fit of $\Delta\sigma/\sigma_{\mathrm{D}} = \Delta G/G_{\mathrm{D}}$ where σ_{D} and G_{D} are the Drude conductivity and conductance. Since k_F is determined independently from Shubnikov–de Haas measurements, $\sigma_{\mathrm{D}} = k_F l_e$ determines l_e. For samples $A/B/C$, $W = 170/270/230$ nm and $l_e = 220/250/360$ nm, independent of temperature as expected. This also shows that the networks are indeed in the diffusive regime. In sample A the number of transverse channels per wire is $M = k_F W/\pi = 9$, and the number of effective conducting channels on the scale of a is only $M_{\mathrm{eff}} = Ml_e/a \sim 2$. Results for samples B and C are similar.

At higher temperature where no AAS oscillations are visible, we can nevertheless deduce L_φ and l_e by fitting the MC, knowing the temperature independent values of

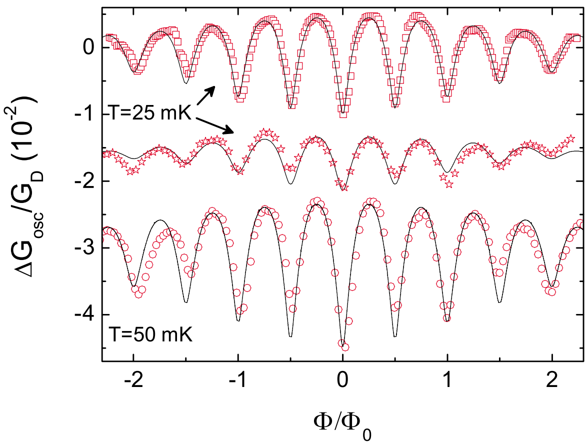

FIG. 4 (color online). Comparison between experiment (symbols) and theory (continuous line) for the oscillating part of the conductance for sample A (□) ($G_D = 3.7 \times 10^{-5}$ S), sample B (★) ($G_D = 5 \times 10^{-6}$ S) with $V_g = -0.15$ V corresponding to $W = 240$ nm and $l_e = 170$ nm, and sample C (○) ($G_D = 2.6 \times 10^{-5}$ S) (shifted down for visibility). The only adjustable parameter is the amplitude of the oscillations.

W and G_D (Fig. 3). Thus a quantitative comparison of L_φ with the theoretical prediction of AAK [7,17] $L_\varphi = \sqrt{2}(\frac{D^2 m^* W}{\pi k_B T})^{1/3}$ written for a 2DEG wire is possible. This theory applies to diffusive metallic wires with a large number of conducting channels in a limit where *e-e* interactions are treated perturbatively. We find a very good agreement (see Fig. 2) which is remarkable for two reasons: (i) we are confronted here with a small number of conducting channels where strong interaction effects could be expected, and (ii) the result of AAK was not extended to network geometry. Recently it was predicted in [18] that L_φ extracted from the AB or AAS oscillations in a single ring of perimeter L should behave like $L_\varphi \propto (LT)^{-1/2}$ corresponding to $R_{12} \sim \exp L^{3/2} T^{1/2}$. This behavior is not observed in our experiment.

For sample B gate voltages between -0.3 and 0.3 V changed the resistance from 30 to 400 kΩ. A good filtering of the gate voltage line is needed to avoid saturation of L_φ. Within our experimental accuracy, we find that L_φ is not changed and still varies as $T^{-1/3}$. We estimated for each gate voltage W and l_e. When the gate voltage varies between 0.15 and -0.15 V, W is unchanged but l_e decreases by a factor of 1.5 and n_e by 30%; G_D decreases by a factor of 5. This shows that the effect of the gate is mainly to disconnect bonds of the network.

As a consistency check we have computed numerically the oscillating part of the MC with formulas (1) and (3) using the value of $W_{\rm eff}$ determined above from the WL envelope of the MC curves. We find that this value also precisely describes the damping of the AAS oscillations, if the oscillations amplitude is multiplied by a factor ranging from 1.6 to 2 depending on the gate voltage. This can be explained by the existence of broken bonds in the network which influences envelope and oscillations differently [19]. We obtain a very good agreement between theory and experiments (Fig. 4).

In conclusion, we have shown that magnetoconductance experiments in GaAs/GaAlAs networks can be described very accurately by the diagrammatic theory of quantum transport in diffusive networks. It is remarkable that this agreement is achieved in a limit where the dimensionless conductance on the scale of the period of the network, Ml_e/a, is of the order 1, and down to temperatures corresponding to $L_\varphi \sim L_T$, i.e., close to the limit of validity of AAK theory. In contrast, metallic wires deep in the diffusive regime have a number of conducting channels of order 1000 and $L_\varphi \gg L_T$ is always fulfilled. Moreover, we extracted from the AAS oscillations the temperature dependence of the phase-coherence length $L_\varphi \propto T^{-1/3}$ that agrees with AAK theory down to 25 mK.

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