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High frequency dynamics and the third cumulant of quantum noise

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Abstract. The existence of the third cumulant S_3 of voltage fluctuations has demonstrated the non-Gaussian aspect of shot noise in electronic transport. Until now, measurements have been performed at low frequency, i.e. in the classical regime $\hbar\omega < eV, k_BT$ where voltage fluctuations arise from a charge transfer process. We report here the first measurement of S_3 at high frequency, in the quantum regime $\hbar\omega > eV, k_BT$. In this regime, experimental results cannot be seen as a charge counting statistics problem any longer. This raises central questions as regards the statistics of quantum noise: (1) The electromagnetic environment of the sample has been proven to strongly influence the measurement, through the possible modulation of the noise of the sample. What happens to this mechanism in the quantum regime? (2) For $eV < \hbar\omega$, the noise is due to zero-point fluctuations and retains its equilibrium value: $S_2 = G\hbar\omega$ with G the conductance of the sample. Therefore, S_2 is independent of the bias voltage and no photon is emitted by the conductor. Is it possible, as suggested from some theories, that $S_3 \neq 0$ in this regime? For these questions, we give theoretical and experimental answers as regards the environmental effects, showing that they involve the dynamics of the quantum noise. Using these results, we investigate the question of the third cumulant of quantum noise in the tunnel junction.

Keywords: mesoscopic systems (experiment), fluctuations (experiment), quantum transport

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Contents

1. Introduction

Physics of current fluctuations has proven, during the last 15 years, to be a very profound topic of electron transport in mesoscopic conductors (for a review, see [1]). Usually, current fluctuations are characterized by their spectral density $S_2(\omega)$ measured at frequency ω:

$$S_2(\omega) = \langle i(\omega) i(-\omega) \rangle, \tag{1}$$

where $i(\omega)$ is the Fourier component of the classical fluctuating current at frequency ω and the brackets $\langle \cdot \rangle$ denote time averaging. In the limit where the current can be considered as carried by individual, uncorrelated electrons of charge e crossing the sample (as in a tunnel junction), $S_2(\omega)$ is given by the Poisson value $S_2(\omega) = e\,I$ and is independent of the measurement frequency ω. At sufficiently high frequencies, however, this relation breaks down and should reveal information about energy scales of the system[1]. In particular, in the quantum regime $\hbar\omega > eV$ (V is the voltage across the conductor), it turns out that the noise cannot be seen as a charge counting statistics problem any longer even for a conductor without an intrinsic energy scale. In this regime, the noise spectral density reduces to its equilibrium value determined, at zero temperature, by the zero-point fluctuations (ZPF):

$$S_2^{(\mathrm{eq})}(\omega) = G\hbar\omega, \tag{2}$$

with G the conductance of the system. Experimental investigations of the shot noise at finite frequency have clearly shown a constant (voltage independent) noise spectral density for $\hbar\omega > eV$ in several systems [2]–[4]. Although these experiments were not able to give

[1] In the following we will not discuss the effect of the electrodynamics of the circuit except when treating the environmental effects. The finite capacitance of a tunnel junction gives rise to a cutoff in the detected noise associated with its RC time (∼8 GHz in our case). This effect is of course important but simply renormalizes the noise as a low pass filter does. Its effect is included in the overall frequency dependent gain of the experimental setup. The dephasing due to this filter is however important in the detection of higher cumulants of noise, including the third one if all the frequencies are high, a case that we do not treat here.

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an absolute value of the equilibrium noise (because of the intrinsic noise of linear amplifiers used for the measurement), one has good reason to believe that ZPF can be observed with such amplifiers. Indeed, it has been proven in other detection schemes, theoretically [5, 6] and experimentally [7]–[10], that ZPF can be detected from de-excitation of an active detector, whereas they cannot be detected by a passive detector which is itself effectively in the ground state.

In view of recent interest in the theory of the full counting statistics (FCS) of charge transfer, attention has shifted from the conventional noise (the variance of the current fluctuations) to the study of the higher cumulants of current fluctuations. Whereas the discrimination between active and passive detector seems to be clear for noise spectral density measurement, the situation is more complex for the measurement of high order cumulants at finite frequency. Indeed, the issues of detection schemes are closely related to the problem of ordering quantum current operators and, if the problem can be solved in a general way for two operators [6, 11], measurements of higher cumulants are pointing to a problem of appropriate symmetrization of the product of n current operators:

$$S_n(\omega) = \langle i(\omega_1)\, i(\omega_2) \cdots i(\omega_n)\rangle\, \delta(\omega_1 + \omega_2 + \cdots + \omega_n). \tag{3}$$

It is the goal of this paper to clearly present the problem of the measurement of a third cumulant in a well defined experimental setup using a linear amplifier as a detector. Until now, measurements of the third cumulant S_3 of voltage fluctuations have been performed at low frequency, i.e. in the classical regime $\hbar\omega < eV,\ k_{\mathrm{B}}T$ where voltage fluctuations arise from charge transfer processes [12]–[14]. We report here the first measurement of S_3 at high frequency, in the quantum regime $\hbar\omega > eV,\ k_{\mathrm{B}}T$. This raises central questions of the statistics of quantum noise, in particular:

(1) The electromagnetic environment of the sample has been proven to strongly influence the measurement, through the possible modulation of the noise of the sample [12]. What happens to this mechanism in the quantum regime?

(2) For $eV < \hbar\omega$, the noise is due to ZPF and keeps its equilibrium value: $S_2 = G\hbar\omega$, with G the conductance of the sample. Therefore, S_2 is independent of the bias voltage and no photon is emitted by the conductor. Is it possible, as suggested from some theories [15]–[17], that $S_3 \neq 0$ in this regime?

In the spirit of these questions, we give theoretical and experimental answers as regards the environmental effects, showing that they involve the dynamics of the quantum noise. We study the case of a tunnel junction, the simplest coherent conductor. Using these results, we investigate the question of the third cumulant of quantum noise.

2. Environmental effects and dynamics of quantum noise

We show in this section that the noise dynamics is a central concept in the understanding of environmental effects on quantum transport. First, we present a simple approach (in the zero-frequency limit) for calculating the effects of the environment on noise measurements in terms of the modification of the probability distribution $P(i)$ of current fluctuations. We do not provide a rigorous calculation, but simple considerations that deal with the essential ingredients of the phenomenon. This allows us to introduce the concept of noise dynamics and determine the correct current correlator which describes it at any frequency.

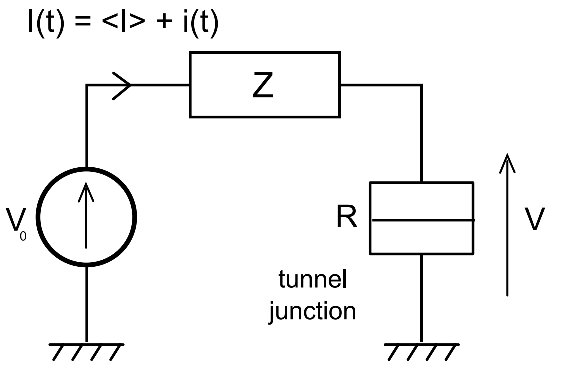

Figure 1. Schematics of the experimental setup. The tunnel junction is connected in series with a voltage source (voltage V_0) through a given impedance Z. Current fluctuations $i(t) = I(t) - \langle I \rangle$ are measured using an ammeter with a bandwidth Δf while the voltage V across the tunnel junction can be measured using a voltmeter.

Then, we report the first measurement of the dynamics of quantum noise in a tunnel junction. We observe that the noise of the tunnel junction responds in phase with the ac excitation, but its response is not adiabatic, as obtained in the limit of slow excitation. Our data are in quantitative agreement with a calculation that we have performed.

2.1. Effects of the environment on the probability distribution $P(i)$

In the zero-frequency limit, high order moments are simply given by the probability distribution of the current $P(i)$ calculated from the current fluctuations measured in a certain bandwidth Δf (see figure 1):

$$M_n = \int i^n \, P(i) \, \mathrm{d}i. \tag{4}$$

The cumulant of order n, S_n, is then given by a linear combination of $M_k \, \Delta f^{k-1}$, with $k \leq n$ [18]. In practice, it is very hard to perfectly voltage bias a sample at any frequency and one has to deal with the non-zero impedance of the environment Z (see figure 1). If V fluctuates, the probability $P(i)$ is modified. Let us call $P(i; V)$ the probability distribution of the current fluctuations around the dc current I when the sample is perfectly biased at voltage V, and $\widetilde{P}(i)$ the probability distribution in the presence of an environment. R is the resistance of the sample, taken to be independent of V and ω. If the sample is biased by a voltage V_0 through an impedance Z, the dc voltage across the sample is $V = Z_\parallel / Z(\omega = 0) \, V_0$ with $Z_\parallel = RZ/(R + Z)$. Even if the sample is in series with the external impedance, we introduce here the parallel combination of R and Z for reasons of convenience, appearing in section 3.2. The current fluctuations in the sample flowing through the external impedance induce voltage fluctuations across the sample are given by

$$\delta V(t) = -\int_{-\infty}^{+\infty} Z(\omega) i(\omega) \, \mathrm{e}^{\mathrm{i}\omega t} \, \mathrm{d}\omega. \tag{5}$$

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Consequently, the probability distribution of the fluctuations is modified. This can be taken into account if the fluctuations are slow enough that the distribution $P(i)$ follows the voltage fluctuations. Under this assumption one has

$$\widetilde{P}(i) = P(i; V + \delta V) \simeq P(i, V) + \delta V \frac{\partial P}{\partial V} + \cdots \tag{6}$$

supposing that the fluctuations are small ($\delta V \ll V$). One deduces the moments of the distribution (to first order in $\delta V = -Zi$) for a frequency independent Z:

$$\widetilde{M_n} \simeq M_n - Z \frac{\partial M_{n+1}}{\partial V} + \cdots. \tag{7}$$

This equation, derived in [19], shows that environmental correction to the moment of order n is related to the next moment of the sample perfectly voltage biased. For $n = 1$ we recover the link between noise and dynamical Coulomb blockade through the noise susceptibility (see below) that appears as $\partial M_2 / \partial V$ in the simple picture depicted here [20]. Let us now apply the previous relation to the third cumulant ($S_n = M_n \, \Delta f^{n-1}$ for $n = 2, 3$):

$$\widetilde{S}_3 \simeq S_3 - 3Z \frac{S_4}{\partial V} \simeq S_3 - 3Z \, S_2 \frac{S_2}{\partial V}. \tag{8}$$

This is a simplified version of the relation derived in [21]. The way to understand this formula is the following: the first term on the right is the intrinsic cumulant; the second term comes from the sample current fluctuations $i(t)$ inducing voltage fluctuations across the sample. These modulate the sample noise S_2 by a quantity $-Zi(t) \, \mathrm{d}S_2/\mathrm{d}V$. This modulation is in phase with the fluctuating current $i(t)$, and gives rise to a contribution to the third-order correlator $\langle i^3(t) \rangle$. This environmental contribution involves the impedance of the environment and the *dynamical response* of the noise which, in the adiabatic limit considered here, is given by $\mathrm{d}S_2/\mathrm{d}V$. However, at high enough frequencies, and in particular in the quantum regime $\hbar\omega > eV$, this relation should be modified to include photo-assisted processes. The notion of the dynamical response of the noise is extended in the following section to the quantum regime in order to subtract the environmental terms properly in the measurement of the third cumulant.

2.2. Dynamics of quantum noise in a tunnel junction under ac excitation

In the same way as the complex ac conductance $G(\omega_0)$ of a system measures the dynamical response of the average current to a small voltage excitation at frequency ω_0, the dynamical response of current fluctuations $\chi_{\omega_0}(\omega)$, that we name *noise susceptibility*, is investigated. This measures the in-phase and out-of-phase oscillations at frequency ω_0 of the current noise spectral density $S_2(\omega)$ measured at frequency ω. In order to introduce the correlator that describes the noise dynamics, we start with those which describe noise and photo-assisted noise. Beside the theoretical expressions, we present the corresponding measurements on a tunnel junction. This allows us to calibrate the experimental setup and give quantitative comparisons between experiment and theory.

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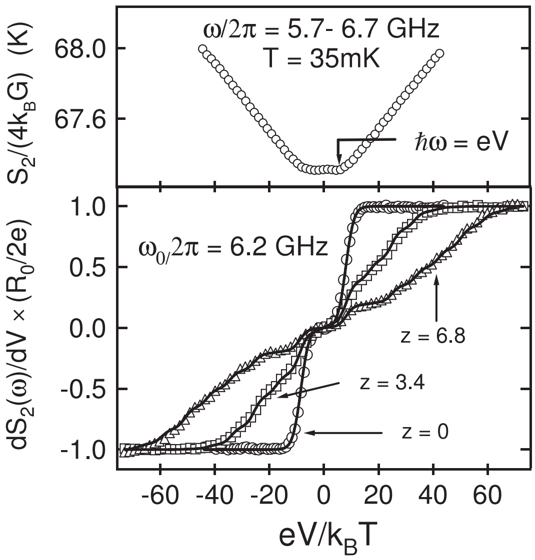

Figure 2. Top: measured noise temperature $T_{\mathrm{N}} = S_2(\omega)/(4k_{\mathrm{B}}G)$ of the sample plus the amplifier with no ac excitation. Bottom: measured differential noise spectral density $\mathrm{d}S_2(\omega)/\mathrm{d}V$ for various levels of excitation $z = e\delta V/(\hbar\omega_0)$. $z \neq 0$ corresponds to photo-assisted noise. Solid lines are fits with equation (10).

Noise and photo-assisted noise. The spectral density of the current fluctuations at frequency ω for a tunnel junction (i.e. with no internal dynamics) biased at a dc voltage V is [1]

$$S_2(V,\omega) = \frac{S_2^0(\omega_+) + S_2^0(\omega_-)}{2}, \tag{9}$$

where $\omega_\pm = \omega \pm eV/\hbar$. $S_2^0(\omega)$ is the Johnson–Nyquist equilibrium noise, $S_2^0(\omega) = 2G\hbar\omega\coth(\hbar\omega/(2k_{\mathrm{B}}T))$ and G is the conductance. At low temperature, the S_2 versus V curve (obtained at point C on figure 3) has kinks at $eV = \pm\hbar\omega$, as clearly demonstrated in our measurement; see figure 2, top. The temperature of the electrons is obtained by fitting the data with equation (9). We obtain $T = 35$ mK, so $\hbar\omega/k_{\mathrm{B}}T \sim 8.5$. Note that a huge, voltage independent, contribution $T_{\mathrm{N}} \sim 67$ K is added to the voltage dependent noise coming from the sample which masks the contribution from the likewise voltage independent ZPF. Indeed, this unknown background noise coming from the amplifier cannot be separated from the ZPF and its estimated value is given with an error on the order of magnitude of ZPF which is here ~0.3 K. When an ac bias voltage $\delta V\cos\omega_0 t$ is superimposed on the dc one, the electron wavefunctions acquire an extra factor $\sum_n J_n(z)\exp(\mathrm{i}n\omega_0 t)$ where J_n is the Bessel function of the first kind and $z = e\delta V/(\hbar\omega_0)$. The noise at frequency ω is modified by the ac bias, to give

$$S_2^{\mathrm{pa}}(V,\omega) = \sum_{n=-\infty}^{+\infty} J_n^2(z) S_2(V - n\hbar\omega_0/e, \omega). \tag{10}$$

This effect, called photo-assisted noise, has been measured for $\omega = 0$ [2]. We show below the first measurement of photo-assisted noise at finite frequency ω. The multiple steps

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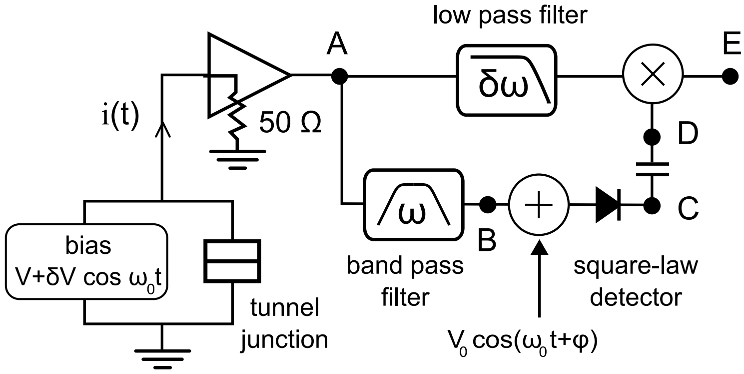

Figure 3. Experimental setup for the measurement of the noise dynamics $X(\omega_0, \omega)$ and the third cumulant $S_3(\omega, \omega_0 - \omega)$ for $\omega \sim \omega_0$. The symbol $\oplus$ represents a combiner, whose output is the sum of its two inputs. The symbol $\otimes$ represents a multiplier, whose output is the product of its two inputs. The diode symbol represents a square law detector, whose output is proportional to the low frequency part of the square of its input.

separated by $eV = \hbar\omega_0$ are well pronounced and a fit with equation (10) provides the value of the rf coupling between the excitation line and the sample δV (see figure 2, bottom).

Noise susceptibility. Photo-assisted noise corresponds to the noise $S_2(\omega)$ in the presence of an excitation at frequency ω_0, obtained by time averaging the square of the current filtered around ω, as in [2] for $\omega = 0$ and in [4] for $\omega \sim \omega_0$. This is similar to the photovoltaic effect for the dc current. The equivalent of the dynamical response of current at arbitrary frequencies ω_0 is the dynamical response of noise at frequency ω_0. It involves the beating of two Fourier components of the current separated by $\pm\omega_0$ expressed by the correlator $\langle i(\omega) i(\omega_0 - \omega) \rangle$. Using the techniques described in [1], we have calculated the correlator that corresponds to our experimental setup, using the 'usual rules' of symmetrization for a two-current correlator and a classical detector. We find the dynamical response of noise for a tunnel junction [20]:

$$X(\omega_0, \omega) = \tfrac{1}{2} \sum_n J_n(z) J_{n+1}(z) \left(S_2^0(\omega_+ + n\omega_0) - S_2^0(\omega_- - n\omega_0) \right). \tag{11}$$

Note the similarity with the expression giving the photo-assisted noise, equation (10). Note however that the sum in equation (11) expresses the *interference* of the processes where n photons are absorbed and $n \pm 1$ emitted (or vice versa), each absorption/emission process being weighted by an amplitude $J_n(z) J_{n\pm1}(z)$.

Experimental setup. The sample is an Al/Al oxide/Al tunnel junction identical to that used for noise thermometry [22]. We apply a 0.1 T perpendicular magnetic field to turn the Al normal. The junction is mounted on a rf sample holder placed on the mixing chamber of a dilution refrigerator. The resistance of the sample $R_0 = 44.2\ \Omega$ is close to 50 Ω, to provide a good matching to the coaxial cable and avoid reflection of the ac excitation. We are at sufficiently low voltage bias to ignore eventual non-linearities in the current–voltage characteristic of the junction. Moreover, non-linearities due to Coulomb

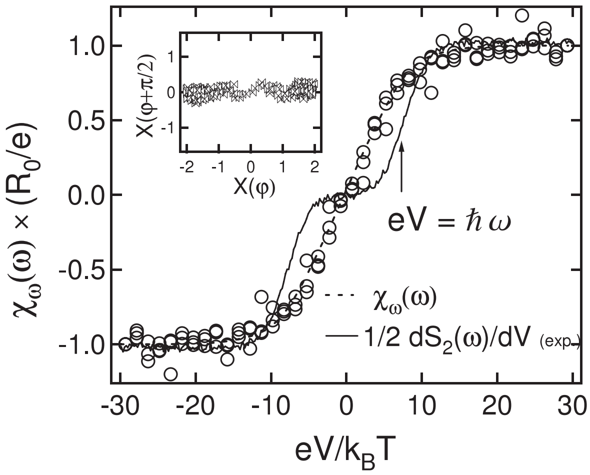

Figure 4. Normalized noise susceptibility $\chi_\omega(\omega)$ versus normalized dc bias. Symbols: data for various levels of excitation ($z = 0.85$, 0.6 and 0.42). Dotted and dashed lines: fits of $\chi_\omega(\omega)$ (equation (13)). Solid line: $(1/2)\,\mathrm{d}S_2/\mathrm{d}V$ (experimental), as a comparison. Inset: Nyquist representation of $X(\omega_0, \omega)$ for $z = 1.7$ (in arbitrary units). The in-phase and out-of-phase responses are measured by shifting the phase φ of the reference signal by 90°.

blockade effects appearing at low temperature and low bias voltage are small (∼0.1%) because of the low resistance R_0 of the sample compared to the quantum of resistance $h/e^2 \simeq 26$ kΩ. This is confirmed by the noise measurement in figure 2. The sample is dc voltage biased, ac biased at $\omega_0/2\pi = 6.2$ GHz, and ac coupled to a microwave 0.01–8 GHz cryogenic amplifier. To preselect the high frequency component $i(\omega)$, we use a 5.7–6.7 GHz band-pass filter (figure 3, lower arm). Its beating frequency ω is shifted to low frequency $\delta\omega$ by using a square law detector and the reference signal $V_0 \cos(\omega_0 t + \varphi)$ in order to mix it with the low frequency component $i(\delta\omega)$. The power detector has an output bandwidth of $\delta\omega/2\pi \sim 200$ MHz, which limits the frequencies ω contributing to the signal: $|\omega| \in [\omega_0 - \delta\omega, \omega_0 + \delta\omega]$. The low frequency part of the current, at frequency $\omega - \omega_0$, is selected by a 200 MHz low pass filter (figure 3, upper arm).

Experimental results. We could not determine the absolute phase between the detected signal and the excitation voltage at the sample level. However we have varied the phase φ to measure the two quadratures of the signal. We always found that all the signal can be put on one quadrature only (independent of the dc and ac bias; see the inset of figure 4(b)), in agreement with the prediction. In the case of small voltage excitation, we define the noise susceptibility which is for noise the equivalent of the ac conductance for current:

$$\chi_{\omega_0}(\omega) = \lim_{\delta V \to 0} \frac{X(\omega_0, \omega)}{\delta V}. \tag{12}$$

$\chi_{\omega_0}(\omega)$ expresses the effect, to first order in δV, of a small excitation at frequency ω_0 to the noise measured at frequency ω. We show in figure 4 the data for $X(\omega_0, \omega)/\delta V$ at small injected powers as well as the theoretical curve for $\chi_{\omega_0}(\omega=\omega_0)$:

$$\chi_\omega(\omega)=\chi_\omega(0)=\frac{e}{2\hbar\omega}\left(S_2^0(\omega_+)-S_2^0(\omega_-)\right). \tag{13}$$

All the data fall on the same curve, as predicted, and are very well fitted by the theory. The crossover occurs now for $eV\sim\hbar\omega$. $\chi_\omega(\omega)$ is clearly different from the adiabatic response of noise $\mathrm{d}S_2(\omega)/\mathrm{d}V$ (solid line in figure 4). However, in the limit $\delta V\to 0$ and $\omega_0\to 0$ (with $z\ll 1$), equation (13) reduces to $\chi_\omega(0)\sim(1/2)(\mathrm{d}S_2/\mathrm{d}V)$. The factor 1/2 comes from the fact that the sum of frequencies, $\pm(\omega+\omega_0)$ (here ∼12 GHz), is not detected in our setup. This is the central result of our work on noise dynamics: the quantum noise responds in phase but non-adiabatically.

3. Third cumulant of quantum noise fluctuations

3.1. Operator ordering

A theoretical framework for analyzing FCS has been developed in [23], for evaluating any cumulant of the current operator in the zero-frequency limit. In order to analyze frequency dispersion of current fluctuations it is necessary to go beyond the usual FCS theory [15]–[17]. An essential problem in these approaches is to know what ordering of current operators $\hat{i}$ corresponds to a given detection scheme. This problem is simpler for S_2: the correlator $S_+(\omega)=\langle\hat{i}(\omega)\hat{i}(-\omega)\rangle$ with $\omega>0$ represents what is measured by a detector that absorbs the photons emitted by the sample, like a photomultiplier. The correlator $S_-(\omega)=\langle\hat{i}(-\omega)\hat{i}(\omega)\rangle=S_+(-\omega)$ represents what the sample absorbs, and can be detected by a detector in an excited state that decays by emitting photons into the sample. Finally a classical detector cannot separate emission from absorption, and measures the symmetrized quantity

$$S_2^{\mathrm{sym.}}(\omega)=\frac{\langle\hat{i}(\omega)\,\hat{i}(-\omega)\rangle+\langle\hat{i}(-\omega)\,\hat{i}(\omega)\rangle}{2}. \tag{14}$$

However, according to the Kubo formula, $S_+(\omega)-S_-(\omega)=G\hbar\omega$, so all these contributions have identical voltage and temperature dependences, at least for a linear conductor. In contrast, different orderings of three current operators give rise to very different results. The prediction for the Keldysh ordering, which is supposed to correspond to a classical detection, is $S_3(\omega,\omega')=e^2I$, independent of frequency even in the quantum regime. As far as we know there is no clear interpretation of this ordering in terms of absorption and emission of photons. We give below two detection schemes for the measurement of $S_3(\omega,0)$ that may lead to different results.

3.2. Measurement of $S_{v^3}(\omega,0)$

Experimental setup. We have investigated the third cumulant $S_{v^3}(\omega,0)$ of the voltage fluctuations of a tunnel junction in the quantum regime $\hbar\omega>eV$. For technical reasons (the input impedance of the rf amplifier is fixed at $Z=50\ \Omega$), we measured voltage fluctuations $v(t)$ instead of current fluctuations $i(t)$. They are related through $v(\omega)=Z_\parallel[i(\omega)+i_{\mathrm{N}}(\omega)]$ where i_{N} is the noise source associated with the amplifier. Note that from the point of view of the intrinsic noise source of the sample, the impedance of

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the environment Z (here, that of the amplifier) and the resistance R of the sample are in parallel. We use the same experimental setup and sample as for the noise dynamics measurement, the only change is that the ac excitation is switched off: $\delta V = V_0 = 0$ (see figure 3). Thus only the noise of the amplifier can modulate the noise of the sample. A 5.7–6.7 GHz band-pass filter followed by a square law detector allows us to mix high frequency components $v(\omega)\, v(-\omega-\delta\omega)$ which are multiplied by low frequency components selected by a 200 MHz low pass filter; we end up with a dc signal proportional to $S_{v^3} \propto \langle v(\omega)\, v(-\omega-\delta\omega)\, v(\delta\omega)\rangle$. The fact that the same setup is used to detect S_3 and χ is quite remarkable: it clearly indicates that the environmental effects in S_3 are indeed described by χ and not by $\mathrm{d}S_2/\mathrm{d}V$.

Experimental results. S_{v^3} at $T = 35$ mK is shown in figure 5(a); these data were averaged for four days. These results are clearly different from the voltage bias result because of the environmental contributions. As described before (see section 2), the noise of the sample is modulated by its own noise and by the noise of the amplifier $S_{2,N}$, to give rise to an extra contribution to S_{v^3}. By generalizing the expression (8), we find, assuming real, frequency independent impedances to simplify the expression (but we used the full expression for the fits of the data),

$$S_{v^3}(\omega,0) = -Z_\parallel^3 S_3(\omega,0) + Z_\parallel^4\,(S_{2,N}(0)+S_2(0))\,\chi_0(\omega) + Z_\parallel^4\,(S_{2,N}(\omega)+S_2(\omega))\,\chi_\omega(0) + Z_\parallel^4\,(S_{2,N}(\omega)+S_2(\omega))\,\chi_\omega(\omega). \tag{15}$$

To properly extract the environmental effects, we fit the data obtained at different temperatures (35 mK, 250 mK, 500 mK, 1 K, 4.2 K). The parameters $Z_\parallel(0)$, $Z_\parallel(\omega)$, $S_{2,N}(0)$ and $S_{2,N}(\omega)$ that characterize the environment only are independent of the temperature of the sample, whereas $S_2(V)$ and $\chi(V)$ depend both on voltage and on temperature. This allows for a relatively reliable determination of the environmental contribution. We have performed independent measurements of these parameters and obtained a reasonable agreement with the values deduced from the fit. However more experiments are needed with another, more controlled environment to confirm our result. The intrinsic S_3 in the quantum regime, obtained after subtraction of the environmental contributions, shown in figure 5, seems to confirm the theoretical prediction of [16, 17] (solid line), i.e. $S_3(\omega,0) = e^2 I$ even for $\hbar\omega > eV$.

4. Conclusion

We have performed the first measurement of the noise susceptibility, in a tunnel junction, in the quantum regime $\hbar\omega \sim \hbar\omega_0 \gg k_{\mathrm{B}}T$ ($\omega/2\pi \sim 6$ GHz and $T \sim 35$ mK) [4]. We have observed that the noise responds in phase with the excitation, but not adiabatically. Our results are in very good, quantitative agreement with our prediction based on a new current–current correlator $\chi_{\omega_0}(\omega) \propto \langle i(\omega) i(\omega_0-\omega)\rangle$. Using the fact that the environmental contributions to S_3 are driven by χ, we have been able to extract the intrinsic contribution from a measurement of $\langle v^3\rangle$ on a tunnel junction in the quantum regime. Our experimental setup is based on a 'classical' detection scheme using linear amplifiers (see figure 6(a)) and the results are in agreement with theoretical predictions: $S_3(\omega,0)$ remains proportional to the average current and is frequency independent [16, 17]. This result raises the intriguing

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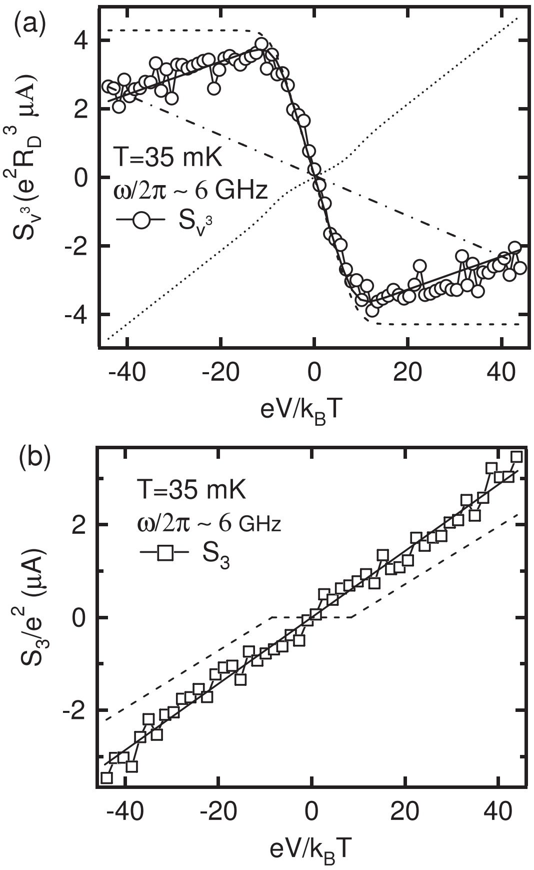

Figure 5. (a) Measurement of $S_{v^3}(\omega, 0)$ versus bias voltage V. The circles are the experimental data. The solid line corresponds to the best fit with equation (15). The dash–dotted line is the perfect bias voltage (i.e., intrinsic) contribution; the dotted line is the contribution of the sample's noise modulated by the sample itself, $\sim S_2\chi$; the dashed line is the contribution of the sample's noise modulated by the amplifier noise, $\sim S_{2,N}\chi$. (b) $S_3(\omega, 0)$ versus bias voltage V (squares) deduced from (a) after subtraction of the environmental contributions. The dashed line is the naive expectation for a detector sensitive to emission only, as in figure 6(b).

question of the possibility of measuring a non-zero third cumulant in the quantum regime $\hbar\omega > eV$, whereas the noise $S_2(\omega)$ is the same as at equilibrium, and given by the zero-point fluctuations. Note that the environmental effect may give rise to a plateau for $eV < k_{\mathrm{B}}T$. We think our determination of the environmental parameters is reliable enough to rule this out in the data presented here. However, we will perform more measurements with a different, more controlled environment to make sure that the intrinsic $S_3(\omega, 0)$ does indeed grow perfectly linearly with voltage even at low voltage.

One can think of another way to measure $S_3(\omega, 0)$ with a photodetector (sensitive to photons *emitted* by the sample), as depicted in figure 6(b). In this case S_3 is the result of correlations between the low frequency current fluctuations and the low frequency fluctuations of the flux of photons of frequency ω emitted by the sample. Since no photon of frequency ω is emitted for $eV < \hbar\omega$, the output of the photodetector is zero and

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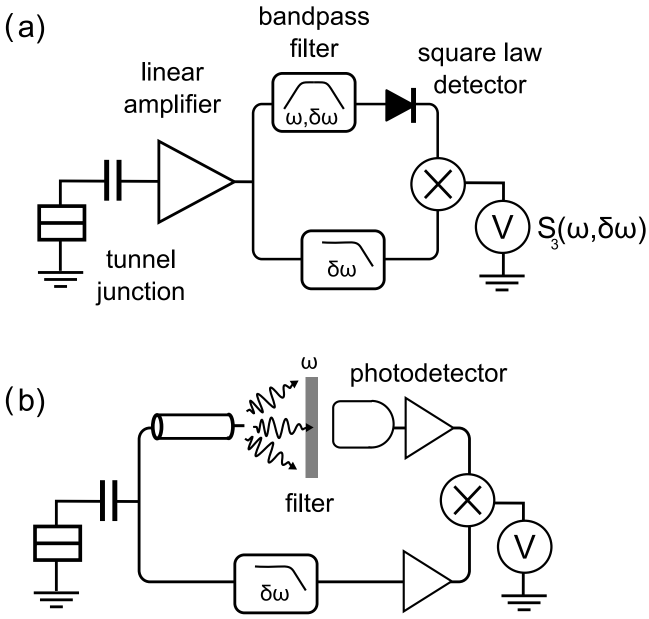

Figure 6. (a) Experimental detection scheme. The symbol ⊗ represents a multiplier, whose output is the product of its two inputs. The diode symbol represents a square law detector, whose output is proportional to the low frequency part of the square of its input. $S_3(\omega, \delta\omega \to 0)$ is given by the product of the square of the high frequency fluctuations with low frequency fluctuations. (b) Equivalent detection scheme using a photodetector to measure the square of the high frequency fluctuations.

$S_3(\omega, 0) = 0$. The expectation of such a measurement is sketched as a dashed line in figure 5(b). Note that such a detection scheme has already been applied to laser diodes [24, 25], though for totally different energy scales.

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