Full counting statistics of avalanche transport: An experiment

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We report the measurement of higher order cumulants of the current fluctuations in an avalanche diode with a stationary dc current. Such a system is archetypal of devices in which transport is governed by a collective mechanism, in this case charge multiplication by avalanche. We have measured the first five cumulants of the probability distribution of the current fluctuations. We show that the charge multiplication factor is distributed according to a power law that is different from that of the usual avalanche below breakdown, when avalanches are well separated.

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I. INTRODUCTION

The variance of the current fluctuations, often simply called “current noise,” is the simplest measure of the statistical aspect of electronic transport in a conductor, beyond the dc current. Its study as a function of other parameters (voltage, temperature, etc.) has been a powerful way to check our understanding of the conduction process in many systems and a tool to obtain information that is hidden in the mean current.1

In order to probe the statistics of the conduction in depth, a knowledge of the distribution function $P$ of the fluctuating current $I(t)$ is necessary, beyond the average $\langle I \rangle$, the dc current, and the variance $\langle I^2 \rangle$ with $I(t) = I(t) - \langle I \rangle$. The brackets $\langle \cdot \rangle$ denote average over the distribution $P$. Most of the time, the full measurement of $P(i)$ is not possible, but a finite number of the moments $\langle i^n \rangle$ of the distribution can be measured, which give some insight into the statistics of $i$. For example, the third moment $\langle i^3 \rangle$ reveals the asymmetry (skewness) of the distribution around the average.

The full counting statistics of charge transfers has been calculated in a quantum conductor driven out of equilibrium by a dc voltage (the so-called shot noise).2 It has triggered the measurement in such systems of higher moments (beyond the second) of current fluctuations.3 This measurement and later work4 have confirmed the Poissonian aspect of transport in a tunnel junction, by proving that the spectral density of the third moment of the current fluctuations is $S_3 = e^2 \langle I \rangle^2$ (where $e$ is the electron charge), independent of temperature. After that, measurements of higher moments have been achieved in Coulomb blockaded quantum dots.5,6

In all these measurements, the statistics of transport is driven by the finite rate at which electrons can pass a barrier.

In this article we report the measurement of higher order cumulants of current fluctuations in avalanche diodes. In such samples, the process that is responsible for current fluctuations is not the transport at the one electron level, but arises from a collective phenomenon, charge multiplication. This occurs due to spontaneous creation of electron-hole pairs in semiconductors in the presence of a high electric field, which strongly accelerates the charge carriers (electrons or holes). When their kinetic energy reaches the gap of the semiconductor, they may give part of it to create a new electron-hole (e-h) pair which, in turn, will be accelerated and give birth to other e-h pairs. Thus, each charge entering the sample generates a current pulse of total charge $Me$ with $M \gg 1$. This avalanche is a statistical process, since the e-h pair creation occurs only with a finite probability per unit time: $M$ is not a well defined number but has a very wide statistical distribution. This distribution has been studied both theoretically and experimentally in the regime where no current is injected but individual e-h pairs are photocreated, giving rise to well separated current pulses.7-9 In such a regime, adequate for the use of avalanche process as an amplifier of individual events, like in the avalanche photodiode, the statistics of the pulses determines the noise added by the detector. We are not considering here this regime which is not stationary; our goal is to investigate the current noise due to avalanche process in the so-called breakdown regime, when the sample sustains a stationary (constant) dc current. Semiconductors working in this regime are used as noise sources.

II. EXPERIMENTAL SETUP

The sample we have studied is a SM-1 avalanche diode manufactured by Micronetics10 (we have measured several devices, all showing very similar features). The avalanche starts when the component is reverse biased by a voltage $\pm 8$ V. Too close to this value there is no stationary regime. We will focus only on the bias region that corresponds to a dc current $I \approx 0.46$ mA where we observe a stationary regime (in the following $I = \langle I \rangle$ stands for the dc current). The schematics of the biasing and measurement circuit is depicted in Fig. 1(a). We use a bias tee to separate the dc part of the current (up to 100 kHz) from its fluctuations (100 kHz to 1 GHz). The sample is biased by applying a dc voltage through a $R_0 = 1$ kΩ resistor in series with the inductor of the bias tee. The fluctuating voltage across the sample is detected through the capacitive part of the bias tee by a spectrum analyzer (at point A) or amplified (in the range 1–500 MHz), filtered and converted into a digital signal (at point D) by a 14 bit, 200 MS/s A to D converter. All the measurements have been performed at room temperature.

A spectrum analyzer measures the power spectral density of a signal as a function of frequency. This quantity is proportional to the noise generated by the sample at frequency $f$ within a bandwidth of 1 Hz. A spectrum for a bias of
I=1 mA is shown in Fig. 1(b). Three regions can be distinguished. At low frequency, the noise spectral density decays with increasing frequency, approximately like 1/f. At higher frequency, the spectrum is almost white, up to ~500 MHz. In order to remove the contribution of the 1/f noise (unrelated to the physics of avalanche) and in order to avoid aliasing, we have used a 25–100 MHz bandpass filter before digitizing the signal [indicated by vertical dashed lines in Fig. 1(b)].

In our setup, the voltage $v_D(t)$ recorded by the digitization card is related to the intrinsic current source $i(t)$ that models the avalanche process by $v_D(t)=H(f)i(t)$ where $v_D(t)$ and $i(t)$ are the Fourier components at frequency $f$ of $v_D(t)$ and $i(t)$. The transfer function $H(f)$ contains the effects of the impedances and gains of the external circuit (bias tee, amplifier, filter) as well as the parasitic impedances inside the component. We have measured the frequency response of the setup as well as the complex impedance $Z_c$ of the sample as a function of frequency and dc voltage with a vector network analyzer in the range 1–1000 MHz. We observe that the sample can be well modeled by a $R_s=4.6$ $\Omega$ resistor in series with a parallel $RLC$ resonator [see Fig. 1(c)]. $R_s$, $L$, and $C$ are independent of the bias voltage, but $R$ varies between 250 $\Omega$ and 750 $\Omega$ when $I$ varies between 0.46 and 4.39 mA, which implies that $H(f)$ depends on the bias voltage too. One also has to know where to incorporate the current source $i(t)$ created by the avalanche in the schematics of the sample. Knowing that the avalanche itself has a white spectrum, we conclude from the frequency dependence of the noise spectral density [Fig. 1(b)] that $i(t)$ must be in parallel with $R_s$ [see Fig. 1(c)]. The high-frequency roll-off observed on the power spectrum comes from the $RLC$ circuit $(1/(2\pi fLC)=600$ MHz). Thus, a direct determination of $i(t)$ could be obtained by dividing by $R$, the voltage across the diode, measured with a high impedance voltmeter. We have used an amplifier with 50 $\Omega$ input impedance and calculated $H(f)$.

FIG. 1. (a) Schematics of the experimental setup. $Z_m$ is the input impedance of the amplifier, that can be modified. (b) Spectral density of voltage fluctuations at point $A$ vs frequency for $I=1$ mA. (c) Equivalent circuit of the sample. $R_s=4.6$ $\Omega$, $L=12.6$ nH, $C=5.1$ pF, and $B[I=1$ mA$]=500$ $\Omega$. $i(t)$ is the current source that models the avalanche.

FIG. 2. Noise spectral density $S_2(f)\equiv\langle|i(f)|^2\rangle$, vs dc current (symbols: measurement; solid line: fit $I^{1.21}$). Inset: measured probability distribution of $v_D$ divided by a Gaussian of same width $\sqrt{\langle v_D^2 \rangle}=32$ mV for $I=1$ mA (indicated by a horizontal line).

(which includes the finite input impedance of the amplifier) to deduce $i(f)$ from $v_D(f)$.

III. SECOND MOMENT

We have recorded the voltage fluctuations $v_D(t)$ and made histograms of them for many bias points, see inset of Fig. 2. The variance $\langle v_D^2 \rangle$ (here the brackets mean time averaging) is related to the noise spectral density $S_2(f)=\langle|i(f)|^2\rangle$ by:

$$\langle v_D^2 \rangle = \int_{-\infty}^{\infty} |H(f)|^2 S_2(f) df.$$

Since we observe that $S_2(f)$ is reasonably frequency independent within the detection bandwidth [see Fig. 1(b)], the variance of $v_D$ and the spectral density of the current fluctuations $S_2$ are simply proportional: $\langle v_D^2 \rangle = H^2 S_2$, where $H(V)=\int |H(f)|^2 df$. We show in Fig. 2 the variations of $S_2$ with the dc current running through the sample. Note that the amount of noise emitted by the avalanche diode is huge, so that the contribution of the amplifier is negligible. For $I=1$ mA, $S_2=1.15 \times 10^{-15}$ $A^2$/Hz which is equivalent to the Johnson noise of a 50 $\Omega$ resistor at a temperature of $18 \times 10^6$ K, to compare with the 300 K noise temperature of the amplifier. Defining the Fano factor $F_2$ by $S_2=F_2 I$, we find $F_2=6.8 \times 10^6$ for $I=1$ mA. The noise is thus extremely super-Poissonian ($F_2=1$ for Poisson noise).

IV. THIRD MOMENT AND ENVIRONMENTAL EFFECTS

From the histograms of $v_D(t)$ we have calculated the third moment $\langle v_D^3 \rangle$. It is related to the spectral density $S_3(f_1,f_2,f_3)\equiv\langle[i(f_1)i(f_2)i(-f_1-f_2)]\rangle$ of the third moment of $i(t)$ by:

$$\langle v_D^3 \rangle = \int |H(f_1)|H(f_2)H(-f_1-f_2)S_3(f_1,f_2,df_1,df_2).$$

If we suppose that like $S_2$, $S_3$ is frequency independent, one has:

$$\langle v_D^3 \rangle \approx H^3 S_3$$

with $H=\int |H(f)|^2 df$. If $S_3$ is frequency independent, as it has been demonstrated experimentally and well understood, there is an additional contribution to $S_3$, given by:

$$\langle v_D^3 \rangle = R_{eff}^2 S_3,$$

$R_{eff}$ is an effective resistor that contains the impedance of the sample and that of the environment.
here the internal RLC circuit and the input impedance of the amplifier \( Z_m \). To estimate the environmental contribution, we have modified \( R_{eff} \) by adding resistances to ground at the input of the amplifier, thus varying \( Z_m \) in the range 4–50 \( \Omega \). This had no effect on \( S_3 \) (besides an overall reduction due to voltage division by the added resistances) except at very low current, where both \( S_3 \) and \( |dS_2/dV| \) are maximum, and where we observe a 5% negative deviation, in agreement with the prediction. Thus, environmental contributions to \( S_3 \) are negligible, i.e., we have access to the intrinsic third moment of the current fluctuations. Moreover, \( S_3 \) (and higher moments) not being influenced by \( Z_m \) demonstrates that the statistics of the current is not affected by the nonlinearity of the component (which is small since \( L \omega \approx R \) in our detection range). We have plotted the result for \( S_3(i) \) in Fig. 3, as \( F_3=S_3/(e^2I) \) vs \( F_2=S_2/(eI) \). We discuss the interpretation of this result later.

V. HIGHER ORDER CUMULANTS

Any statistical measurement has to face a fundamental problem: the central limit theorem, according to which the distribution of the sum of \( N \) independent variables tends to a Gaussian when \( N \) goes to infinity, regardless of the distribution law of each of the variables. In our measurement, a current \( I=1 \) mA integrated during 5 ns corresponds to \( N=3\times10^7 \) electrons, and the measured histograms are very close to gaussians; see inset of Fig. 2. Nevertheless one can access the probability distribution \( P(i) \) by working with its cumulants, defined as follows. We introduce the characteristic function \( \chi(z)=(e^{zi}) \) (\( i \) is the fluctuating current). The Taylor expansion of \( \chi(z)=\sum_n(i^n)z^n/n! \) provides the moments \( \langle i^n \rangle \). The cumulants \( \langle \langle i^n \rangle \rangle \) are defined by the Taylor expansion of \( \ln \chi(z)=\sum_n\langle\langle i^n \rangle\rangle z^n/n! \). If \( P(i) \) is Gaussian, \( \ln \chi(z) \) is a second degree polynomial, and \( \langle\langle i^n \rangle\rangle=0 \) for \( n \geq 3 \). Thus the cumulants reveal how much a distribution deviates from a Gaussian. From the histograms of \( v_p(i) \) we calculate the cumulants \( \langle\langle v_p^2(i) \rangle\rangle \) from which we deduce the spectral cumulants of \( P(i) \), which we note \( C_n \), supposing that they are frequency independent, as \( C_2=S_2 \) and \( C_3=S_3 \). A simple calculation gives indeed: \( \langle\langle v_p^2(i)\rangle\rangle=H_nC_n \) where the coefficients \( H_n \) are given by: \( H_n=\int_0^\infty H^n(t)dt \) with \( H(t) \) the inverse Fourier transform of \( H(f) \), i.e., the impulse response of the circuit. This definition coincides with that given before for \( n=2 \) and \( n=3 \). In order to check the calibration coefficients \( H_n \), we have measured the first five cumulants of a pseudorandom current generated by a 1 GS/s arbitrary waveform generator with a statistics that we have computed. The ratio of the measured cumulants to the real ones provides us with another set of calibration coefficients \( H'_n \). We obtain \( H_n \approx H'_n \) up to \( n=5 \) (see inset of Fig. 4). The systematic error on the sixth spectral cumulant is only a factor of 2. We conclude that we can perform a reliable and quantitative measurement of \( C_n \) up to \( n=5 \). We have measured the first five spectral cumulants of the current noise of an avalanche diode. We observe that they all decrease as a function of the average current as a noninteger power law (see, e.g., \( S_2 \) on Fig. 2). In order to synthesize our results, we show on Fig. 3 the Fano factors \( F_n \) of the \( n^{th} \) cumulant (\( n \approx 3 \)) as a function of the second one \( F_2 \); we define the (dimensionless) Fano factors by: \( C_n=F_n e^{n-1}I \). We observe a remarkable behavior: \( F_n(i) \approx F_2^{n-2}(i) \), which can be rewritten as \( F_n(i)=g_s G^n(i) \) (\( n \approx 2 \)) where the function \( G(i) \) is the same for all the cumulants and \( g_s \) are constants. Note that all the cumulants can be in principle calculated from the data, but the uncertainty on \( C_n \) due to the statistical aspect of the measurement (we record \( v_p(i) \) during a finite time) grows fast with \( n \). Figure 3 corresponds to \( 8 \times 10^{13} \) independent measurements of the current for each bias point.

VI. INTERPRETATION

The result of Fig. 3 is central in the understanding of the statistical aspect of transport in the avalanche regime. A Poisson distribution with an effective charge \( q=Me \) has Fano factors \( F_n=M^{n-1}F_2^{n-1} \), even if the effective charge depends on \( I \). Our measurements clearly rule out this scenario. The reason for this is that the avalanche itself is responsible for
the current noise. We will now compare our experimental result with what is predicted in the nonstationary regime. An injected charge gives rise to \( m \) charges with a power-law distribution \( P(m) \). Thus, an injected photocurrent \( i_0 \) gives rise to a dc current \( I = \langle m \rangle i_0 \) and to current fluctuations of spectral cumulants \( C_n = e^{n-1} i_0 \langle m^n \rangle \). The multiplication factor \( m \) has been shown to obey a probability distribution such that: \( \langle m^n \rangle \sim \langle m \rangle^{2n-1} \). This implies for the Fano factors: \( F_n = \langle m \rangle^{2n-2} \approx F_2^{-1} \), in strong contrast with our results.

The nonstationary regime is a particular case of avalanches, which are more generally described by a probability distribution that is a power law, \( P(m) \approx m^\alpha \), which reflects how the probability distribution different from what has been reported until now in the nonstationary regime, and different from what is predicted using mean-field approximation in avalanche theories. This may open new routes for the study of other components, or complex conductors such as those with charge-density waves. Moreover, avalanche diodes are advertised as perfect Gaussian noise sources. This is definitely incorrect and may have practical consequences. For example, using the sign of the instantaneous fluctuating current as a source of random bits gives, for the sample we have analyzed and in our bandwidth, \( P(i > 0) \approx 1.01 P(i < 0) \). To cure this, the central limit theorem may help, with the price of a smaller bit rate.

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13. We have calculated the correlation function \( \langle v_D(t) v_D(t+\tau) \rangle \), proportional to the Fourier transform of \( S_{\Delta}(f_1,f_2) \). It is very peaked at \( \tau_1 = \tau_2 = 0 \), which means that \( S_3 \) is indeed frequency independent.