Current dependent fluctuations in a Bi$_2$Sr$_2$CuO$_{6+\delta}$ thin film

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The current dependence of the excess conductivity is measured up to $\simeq 3T_c$ for a Bi$_2$Sr$_2$CuO$_{6+\delta}$ thin film, as a function of doping. It is found to be anomalously sensitive to the transport current and to behave as a universal function of $T/T_c$ in the whole doping range. We discuss these results in the perspective of a granular superconductor with a gapless-like behavior.

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I. INTRODUCTION

Superconducting fluctuations are strongly reduced when the out of equilibrium superfluid velocity due to a transport current reaches a critical value, $\Delta/|\mu|$, similar to the depairing velocity obtained in the superconducting regime. Using the time dependent Ginzburg Landau theory, Schmidt and Hurault computed the associated critical electrical field in the case of Gaussian fluctuations[1–3]. Improvements for layered materials[4, 5] or taking into account the critical regime close to the superconducting transition temperature were obtained later[6, 7]. Clear experimental evidence for the validity of the theories in the Gaussian regime were brought in the case of bulk conventional superconductors[8–11]. However, similar studies on high-$T_c$ superconductors are rare, principally due to the experimental difficulty to reach higher critical electrical fields in these materials[12–14]. In Ref. [14], an anomalously large sensitivity of the superconducting fluctuations to the transport current was pointed out for Bi$_2$Sr$_2$CuO$_{6+\delta}$ (Bi-2201), which results in an apparent characteristic electrical field several orders of magnitude lower than would be expected from a simple estimate for this material. The origin of this discrepancy is still unclear[14]. Among possible explanations, the existence of microscopic disorder, at a length scale smaller than the one of the superconducting fluctuations, was proposed. In this contribution, we explore further the effect of the transport current for a Bi-2201 thin film, extending the non-linearity measurements up to $T \simeq 3T_c$ and from overdoped to underdoped superconducting states. We discuss the results in the perspective of a granular material.

II. EXPERIMENTS

A single crystal, c-axis oriented, Bi-2201 thin film was grown epitaxially (Fig. 1, inset) on a heated SrTiO$_3$ substrate, by reactive rf sputtering with an oxygen rich plasma (Ref. [15] and Refs therein). It consists of grains with a c-axis perpendicular to the film, with sharp twin boundaries at the atomic level and no phase shift between them, due to the orientation imposed by the SrTiO$_3$ substrate. X-ray diffraction analysis also allowed us to check the absence of parasitic phases, to the accuracy of the diffraction spectra, i.e. about 3% (Fig. 1). Resistive measurements, which are sensitive to the presence of superconducting intergrowth phases appearing as a kink in $dR/dT$ curves, did not show any of these. This is expected in the case of Bi-2201 for which no such phases are observed. After deposition of 2700 °A thick material, Au contacts were sputtered onto the sample, which was patterned in the four contact transport geometry, with a current carrying strip of width and length 100 µm and 130 µm respectively. The orientation of the strip was such that the current flew along the CuO bond direction. Doping was varied by changing the oxygen content of the film. The film was annealed for one hour at 270 °C under the appropriate oxygen pressure. The sample resistance was monitored, allowing us to characterize in situ the variation of the sample doping level. The resistance rapidly stabilized and stayed constant, thus insuring the thermal equilibrium of the oxygen content. The sample was then rapidly quenched to the ambient temperature. Doing so, we obtained different doping levels for the same sample, with a 10% – 90% resistive transition about 2 K wide. The maximum superconducting temperature, measured at the mid-point of the transition, was $T_c = 19.9$ K.

The sample resistance at vanishing current density was measured using a lock-in detection with a current of 10 µA. Larger current resistance measurements, below $I = 30$ mA (current density $J = 1.2 \times 10^6$ A cm$^{-2}$), were performed using the pulse-probe technique described in Ref. [14]. The current was fed into the sample during a 10 µs pulse and, 1 µs later, a probe pulse with a lower current $I_0 = 2$ mA and negligible Joule heating was used to measure the sample resistance and its temperature, while the thermal relaxation since the main pulse is negligible. The repetition rate was $10^{-4}$. The temperature increase determined in this way was less than 0.3 K for the largest current value. The measured non linearity due to the electronics of the experimental setup was 0.3% for the higher current. Such a value is not negligible as compared to the non linearity of the sample conductivity. As a consequence, a correction was made for the sample resistance, using exactly the same procedure for
all data shown below. It consisted in a normalization of the data, so that the corrected sample resistance was independent of the current in the range 110 K – 120 K. We note that this procedure may eliminate the non linearity which may be present at temperature higher than this range. However, the non linearity uncovered by this procedure being strongly increasing with decreasing temperature, this validates a posteriori the low temperature data. The heating of the sample is a major problem in these experiments. In our case, the transport current needed to suppress the fluctuations is relatively small, as compared to other high-Tc samples [14, 16, 17], so that the sample temperature rise is also small. As a consequence, we are clearly not in the situation where the superconducting transition appears as shifted due to heating by several Kelvin [16, 17]. Also, the last temperature decrease at the end of the main pulse - which cannot be measured by our technique and is set by the thermal resistance at the film/substrate interface and by the dissipated power per unit area (70 W/cm² in our case, as compared to 10⁴ W/cm² for the thinner film in ref. [17]) - is reduced to a few hundredth of Kelvin and could be neglected, while higher power would either require a specific temperature correction [16] or pulses shorter than the film relaxation time. The following results validate a posteriori the procedure used to evaluate the sample temperature, as severe uncorrected heating effects would make the apparent non linear field effect independent of the doping level - which is not observed here - and as a less disordered sample consistently exhibits a reduced non linearity.

III. RESULTS AND DISCUSSION

The superconducting transition temperature for various doping levels is shown in Fig. 2, as a function of the sample conductivity at 250 K, \( \sigma(250 \text{ K}) \). As one has roughly \( \sigma \propto \rho \propto \delta \), where \( \rho \) is the CuO₂ plane hole concentration, the data may be fitted using the parabolic empirical law [18, 19]:

\[
T_c / T_c^{opt} = 1 - C(p - \rho^{opt})^2,
\]

where \( T_c^{opt} \) and \( \rho^{opt} \) are the superconducting transition temperature and hole concentration for optimal \( T_c \). The excess conductivity in the limit of vanishing current density, \( \sigma'(0) \), with respect to the normal-state conductivity as obtained from a fit of the normal-state resistance above 2 \( T_c \) to a power law, is shown in Fig. 3 (the excess conductivity is defined as \( \sigma' = \sigma - \sigma_{\text{normal}} \)). As well known for this procedure, the uncertainty on the excess conductivity is essentially due, on the low temperature side, to the finite transition width and, on the high temperature side, to the uncertainty for the normal-state conductivity. Within these limitations, the excess conductivity for both underdoped and overdoped states is found roughly universal (i.e. dependent on the reduced temperature \( T / T_c \) only) and well described by the two dimensional Aslamazov-Larkin theory for Gaussian fluctuations (A-L) [20] and the high-temperature extension in Ref. [21, 22], with no fitting parameter. Equivalently, the temperature at which the excess conductivity meets some criterion should be proportional to the sample transition temperature only. Taking \( \sigma' = \sigma_0 \), where \( \sigma_0 = e^2 / 16 \hbar s \) is the universal fluctuation conductance, with \( s \) the superconducting CuO₂ plane separation, one may check in Fig. 2 the universal character of the fluctuations on the whole range of doping. The large current measurements (Fig. 4), allow ones to further uncover some weaker fluctuations at higher temperature. Evaluating the temperature at which the excess conductivity for the higher current is reduced by about \( \Delta \sigma' \equiv \sigma'(2m\text{A}) - \sigma'(40 \text{ mA}) = 10^{-1} \sigma_0 \), we find that, within the experimental uncertainty, the fluctuations uncovered by the current are again universal (Fig. 2). We note, in particular, that the current dependent fluctuations do not exhibit any enhancement on the underdoped side of the phase diagram with respect to the overdoped one (Fig. 2). There is a slight asymmetry, which may be due to the fact that our criterion is obtained at constant current, while a constant electrical field criterion would shift the points on the overdoped side – with a lower resistivity – to higher temperatures.

The universality of the current dependent part of the excess fluctuations holds up to a reduced temperature as high as \( T / T_c \approx 3 \), as can be seen in Fig. 5. From the sample resistance variation, \( R(I) - R(0) \), as shown in Fig. 6, one may evaluate the characteristic electrical field for the excess conductivity non linearity, within the Gaussian theory. For \( E \lesssim 0.3 E_c(T) \), one has \( \sigma'(E)/\sigma'(0) \approx 1 - 0.6 (E/E_c) [1-3] \). Taking \( \rho(E) \approx \rho_n \) the normal-state resistivity (one has \( \rho(\rho_n)/\rho_n < 10^{-1} \) for \( \epsilon = (T - T_c)/T_c > 0.2 \), we obtain:

\[
[\rho(E) - \rho(0)]/\rho_n \approx \sigma'(0) - \sigma'(E) \approx 0.6 \sigma_0 e^{-1}(E/E_c)
\]

Then, for weak variations of the sample resistivity with current (so that \( E \propto I \)), one expects \( R(I) - R(0) \propto e^{-5/2} I \), using \( E_c \propto e^{3/2} \). As shown in Fig. 6, we do observe such a linear dependence with the current. However, the temperature dependence is clearly weaker, being close to \( e^{-\alpha} \) with \( \alpha = 1.2 - 1.5 \) (Figs 5, 6). The apparent critical electrical field obtained from the resistance variation is then found increasing with temperature as \( e^{-\} \) (a dependence clearly weaker than the theoretical one, \( e^{-3/2} \)) with a typical value \( E_c(40 \text{ K}) \approx 3 \times 10^5 \text{ V m}^{-1} \) (Fig. 7). This is well below the expected value \( (16 \sqrt{3} k_B T_c / \pi e \xi_0) e^{3/2} \approx 3 \times 10^5 e^{3/2} \text{ V m}^{-1} \) for the two dimensional Gaussian case [1-3], using \( \xi_0 = 60 \text{ Å} [23] \), which extends the results obtained in the interval \( e \lesssim 0.5 \) in Ref. [14] to higher temperatures.

There is little other experimental data on high-\( T_c \) superconductors to compare with our measurements. Non linearity was measured for YBa₂Cu₃O₇+\( \delta \) in Ref. [12]. However, the temperature range (\( e < 0.02 \)) was too narrow to allow for a comparison. In Ref. [13], characteristic electrical field measurements were reported for Bi₂Sr₂CaCu₂O₈+\( \delta \) (Bi-2212) with \( T_c \approx 78 \text{ K} \) in a larger temperature range (\( e < 0.1 \)). The electrical field was found somewhat smaller than expected and a coherence
length value as high as $\xi = 100 - 200$ Å must be used to account for the data. Furthermore, the data is clearly better described as $E_c \propto 1/\xi^2$, rather than by the conventional behavior $E_c \propto 1/\xi^{1/2}$ proposed in Ref. [13] (Fig. 8). Then, although the excess conductivity in Bi-2212 exhibits a smaller electrical field dependence than in Bi-2201, it is still larger than expected (in agreement with the results in Ref. [14]) and the temperature dependence of the effect is found similar to the one described in this paper for Bi-2201.

We shall now discuss these results in the framework of a granularity. As a granular superconductor may exhibit arbitrarily small critical current density, one might also expect that a transport current can reduce superconducting fluctuations more easily than in the bulk. Ideally, such a material consists of identical superconducting grains surrounded by an insulator or a normal metal, so that the grains are coupled through junctions. In the present case, granularity should not be understood as the presence of well defined grains with sharp boundaries, as can be found in conventional granular materials, but as the presence of inhomogeneities or ‘islands’, such as the ones observed in Refs. [24-27]. There has already been several proposals to account for anomalous properties of some high-$T_c$ superconductors — positive curvature of $H_d(T)$, Meissner and Nernst effects above $T_c$ — using granularity [28, 29]. The following considerations all pertain to the case of an s-wave superconductor, whereas Bi-2201 materials is likely a d-wave one. It is known that tunnel junctions from such materials may greatly differ from the s-wave case. However, in the large temperature limit, the phase space around the gap nodes scales with the temperature, so that the d-wave junction should essentially behave as conventional ones [30].

To begin with the vanishing-current resistivity measurements, one may wonder whether the observation of standard universal fluctuations (Fig. 3) is compatible with a disordered material. As noticed in [22], the universality of the fluctuations for a two-dimensional system is robust against all sorts of perturbations, such as impurities or localization and this is likely true also for a two dimensional granular superconductor (see the discussion by Harris for the case of the two-dimensional XY model with disorder [31] and Ref. [32] for a modelization with an array of resistively shunted junction with moderate dissipation — actually isomorphic to the first case, as well as the fluctuation conductivity obtained in Ref. [33] in the 3D case). As an illustration, granular NbN films transport properties were investigated in Ref. [34]. The conductivity was found to behave as $\sigma \propto (T - T_c)\Delta^2$ over one decade, where $T_c$ is the temperature for phase ordering of the superconducting network. This is in complete disagreement with the A-L prediction $\sigma \propto (T - T_c)^{3.5}$. However, a close examination of the data in Ref. [34] shows that there is indeed a temperature regime, $(T - T_c)/T_c < 0.1$, where the A-L result is observed. The high-temperature power law may then correspond to the asymptotics in Refs. [21, 22]. Thus, we may conclude that standard conductance fluctuations can be preserved in a two dimensional granular superconductor (as long as the inhomogeneity is weak enough, so that the superconducting transition is not dominated by percolation [35]), and the present low-current measurements do not rule out granularity in our case.

We now consider the non-linear regime for the fluctuations in the presence of an array of ideal SIS junctions. Below $T_c$, the situation of grains coupled through Josephson junctions was considered in Ref. [36]. It was shown that, for small grains (as compared to the coherence length), the classical solution is identical to the one of a dirty superconductor, provided one uses the effective normal state resistivity, which incorporates the junction resistance. In this case, we would expect a conventional behavior in the fluctuation regime. However, as noticed in Ref. [36], this effective medium result should not be valid when thermal or quantum fluctuations become large enough to destroy phase coherence between grains. In the present case, the normal state resistivity at $T_c$ per square and per superconducting plane is high $(R \simeq 1.6 \text{ k}\Omega$ for the optimally doped state, and $R \simeq 6.4 \text{ k}\Omega$ for the underdoped state with $T_c = 7.5 \text{ K}$), as compared to the critical value $R_c = h/4e^2 = 6.45 \text{ k}\Omega$[37]. Then, considering the additional effect of the charging energy [38], there could be in the present case large fluctuations which contribute to destroy the phase coherence between grains, with a paracohherent state above $T_c$. As a consequence, the classical treatment might not be appropriate here.

The non-linear transport properties above $T_c$ is in this case a largely unexplored field. Kulik derived the expression for the non-linear excess conductivity of a single tunnel junction with negligible charging energy [39]. As expected, when the resistance of the junction is large ($\epsilon \lesssim R/R_c$), tunneling is dominated by thermal fluctuations in the junction. The characteristic voltage across the junction for non linearity is set by the lifetime of the tunneling pairs in the junction. Further above $T_c$, the characteristic voltage is set by the pair relaxation time. In both cases, the temperature sets the energy scale and, apart from different temperature dependences, the result for the characteristic electrical field is essentially the same as for the bulk, $E_c \simeq T_c/\xi$. When coupling is strong enough so that the correlation length of the array exceeds the separation of the grains, the array has collective modes [40]. Assuming a uniform flow, the effect of the transport current is a mere shift of the array free energy, just as a uniform stress on an harmonic crystal leaves its normal vibrating modes unchanged. As a consequence, within the Debye approximation [33], we do not expect that the current should alter the fluctuations of the array before it breaks the coupling in the individual junctions.

Then, it seems impossible to explain a substantial reduction of the characteristic electrical field from a model of SIS junctions array. So far however, we have not considered the possibility that, although granular, the junctions are not of the SIS type, but of the SNS one. In
the latter case, below $T_c$, the order parameter induced in the normal metal by the proximity effect is exponentially reduced with respect to its value in the superconductor [41], as \(\exp(-d/\xi_n)\), with \(d\) the grain separation and 
\[\xi_n = h v_F/2\pi k_B T\] 
the normal metal-coherence length in the clean limit. Besides reducing the transition temperature to the decoupling one[29], we expect that, in the fluctuation regime when \(d > \xi\), the characteristic voltage across the junction should be determined by the effective gap value in the metal, which is reduced roughly in the same proportion (the diffusion time in the normal metal being less than the pair lifetime, \(\pi h/8k_B(T - T_c)\)). Using \(h v_F \simeq 1\) eV Å, one has \(\xi_n \simeq 90\) Å, so that to obtain the measured reduction of the effective gap value, \(\xi \exp(-d/\xi_n)/d\), of two orders of magnitude, normal domains as large as \(d \simeq 200\) Å are needed (we have used the clean limit for these estimates, as the mean free path is found \(l \gtrsim \xi_n\) from the plasma frequency and the resistivity — see below). There are, however, objections to the existence of such normal metal domains. First, such a large value of the domain size is unlikely, being much larger than the inhomogeneities which have been put into evidence in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. Then, while in the case of Bi-based metallic materials domains may exist, as overdoping is easy (a possibility not available in other materials, such as YBa$_2$Cu$_3$O$_{7-\delta}$), we would expect a reduction of the normal domain size when doping is reduced (approaching the insulating state), while the effective gap reduction appears to be uniform with doping.

Several other mechanisms have been proposed that may also contribute to a decrease of the apparent gap value in junctions $I - V$ characteristic. Below $T_c$, these mechanisms are invoked to account for the small $I, R_n$ product generally observed for grain boundary junctions[42]. As pointed out by Halbritter[43], real junctions in cuprates — either natural or artificial — may be far from the idealized sharp barriers, which are fully described by the energy barrier and the tunneling distance. In particular, he has shown that resonant-states in an insulating barrier may account for a proximity-like effect, but with a much reduced effective normal metal coherence length, which can be in the nanometer range. Identical mechanisms may be at work to weaken the effective superconducting interaction of the fluctuating tunneling pairs. Such mechanisms are needed to reconcile the granular description and the experimental observations.

More generally, it must be noticed that the resonant states mechanism mimics a gapless superconductivity. As stressed in [44], the microscopic mechanism at the interface is not truly gapless, as both the gap value and the transition temperature are reduced at the interface, so that the ratio $\Delta/T_c$ is essentially invariant. However, the granular material made of such junctions does exhibit a reduced $\Delta/T_c$ ratio, when $\Delta$ is inferred from the nonlinear transport properties. One may wonder if a truly gapless superconductivity could account for our results. Gapless superconductivity may be obtained from a bulk scattering mechanism. As well known, in the case of a d-wave superconductor, the superconducting transition temperature, the superfluid density, the depairing current density, all decrease roughly linearly in a large range of the inverse scattering rate, $1/r$, reaching zero for $h/2\tau k_BT_{c0} \simeq 1$[45, 46]. A strong pair-breaking effect is plausible in the case of Bi-2201. An estimate for the scattering rate $\tau(T_c) \simeq 0.05$ ps may be obtained, using $\omega_p = (4\pi ne^2/m^*)^{1/2} \simeq 9000$ cm$^{-1}$ as the plasma frequency [47] and $\rho(T_c) = 190\mu\Omega$ cm. This yields for the pure material $T_{c0} \simeq h/2\tau \simeq 10^2$ K and $T_c/T_{c0} \simeq 0.2$. We expect that such a disordered, gapless superconductor may exhibit a strongly reduced characteristic electrical field, as this field is determined by the energy spectrum of the excitations, rather than by the non-zero pair potential[48]. However, scanning tunneling spectroscopy showed clear evidence for a superconducting gap in Bi-2201, with well defined coherence peaks, but strongly inhomogeneous at some length scale below 20 nm[49]. As a consequence, a bulk, homogeneous description of the disorder in this material, leading to gapless superconductivity, seems to be inadequate and a granular description more appropriate. However, in the absence of available theories for the non linear excess conductivity in disordered d-wave materials, we cannot totally exclude the homogeneous disordered scenario.

So far, we have considered only the effect of the current on the fluctuating inter-grain Josephson current, but it is worth pointing out the possibility of a contribution of the normal state of the granular medium to the non linear resistivity. Indeed, there is an electric field-induced conduction in the case of granular metals, which has been modeled in refs.[50, 51]. The model accounts for both the exponential decrease of the low temperature conductivity with electric field, and the activated behavior of the low electric field regime when transport is dominated by the charging energy of the grains. However, this picture is difficult to reconcile with the observation that the non-linearity effect follows the same doping dependence as the transition temperature and, thus, is likely related to the superconducting fluctuations rather than to the normal state. This difficulty may be circumvented provided one links the occurrence of superconductivity with the normal state properties themselves. Going to the underdoped regime, one may expect an increase of the charging energy and of the barrier height between grains, yielding an increase of the characteristic electric field [50, 51], consistent with our observations. Moreover, assuming that the superconducting transition is set by the charging energy being of the order of the Josephson coupling energy, it is found that the electric field for non-linearity is reduced by the WKB tunneling rate exponential factor, with respect to the intrinsic value[50, 51]. However, within such a picture, it is still difficult to account for the scaling of the non linear effect with the critical temperature in the whole doping range. In a general way, this scaling indicates that granularity is here likely different from the one observed in [24–27], which
was found more pronounced in the underdoped regime than it is in the overdoped one. Then, although a similar mechanism should not be excluded in the case of an intrinsic granularity (such as a real space electronic phase separation for a slightly doped Mott insulator), disorder should, in the present case, be attributed to some local off-stoichiometry or some structural disorder, independent of the doping level. Also, it is worth to underline that such a disorder apparently brings the energy scale for fluctuations suppression to such a low value, that the pseudo-gap phase for this compound [49] would not be uncovered by the current, even though it could involve superconducting fluctuations. Concerning the origin of these inhomogeneities, amongst the Bi family, Bi-2201 exhibits a more pronounced modulation of the BiO plane[52]. Such a disorder was invoked in Ref. [53] to explain the anomalously low value of the superconducting transition temperature, \( T_{c}^{\text{max}} \approx 20 \text{ K} \), where parent compounds such as \( \text{Tl}_{2}\text{Ba}_{2}\text{CuO}_{6} \) or \( \text{HgBa}_{2}\text{CuO}_{4} \) have a maximum critical transition temperature \( T_{c} \approx 90 \text{ K} \). Moreover, when cation substitution \( \text{La/Sr} \) is made on Bi-2201, the modulation is reduced and the maximum transition temperature is increased[53]. The nanoscale domains arising from this modulation could be at the origin of some granularity in the superconducting plane. Although there is at present no direct evidence that these domains induce some inhomogeneity in the superconducting gap[49], it remains that the La substituted material exhibits a reduced sensitivity to the electrical field. This is shown in Fig. 9 where the \( \text{Bi}_{2}\text{Sr}_{1.7}\text{La}_{0.3}\text{CuO}_{6+x} \) material, with a residual resistivity less than the pure material and, presumably, less disorder, also clearly shows a reduced effect of the electrical field.

### IV. CONCLUSION

In conclusion, we have shown that the non-linearity with current of the excess-conductivity amplitude for a \( \text{Bi}_{2}\text{Sr}_{2}\text{CuO}_{6+x} \) thin film at various doping states, measured well above \( T_{c} \), is related to the reduced temperature \( T/T_{c} \) only. Its magnitude as well as its temperature dependence are clearly different from the expectations from the theory of Gaussian fluctuations in a conventional two-dimensional superconductor. Based on scanning tunneling measurements, we suggest that this could be the signature of a granular superconductor, with a gapless-like behavior.

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FIG. 1: X-ray diffraction spectra for the Bi-2201 film. Θ – 2Θ scan. Inset: Φ scan of the (115) reflection of the film.

FIG. 2: Full circles: resistive transition temperature; hollow circles — respectively stars: temperature at which the excess conductivity — respectively the excess conductivity non-linearity — meets a given criterion. Line, parabolic fit for the transition temperature. Dotted lines are homothetic to the full one.
FIG. 3: Excess conductivity in the vanishing current limit vs reduced temperature, in the 2D universal conductance unit, using $s = 12.3$ Å. States are labelled according to Fig. 2 notations (states e and f: overdoped $T_c = 17.4$ K and 12.8 K; state g: underdoped, $T_c = 17.2$ K). The full line is the Aslamazov-Larkin result[20] and the dotted one is the high temperature asymptotics in Ref. [22]. Horizontal error bars originate from the finite transition width and the vertical ones from the uncertainty on the normal state conductivity.
FIG. 4: Excess conductivity non linearity for various currents $I$. Doping states as shown in Fig. 2 (from underdoped, top, to overdoped, bottom).

FIG. 5: Universality of the non-linear excess conductivity. The line slope is $-1.4$. 
FIG. 6: Sample resistance variation, \( dR = R(I) - R(2 \text{ mA}) \), scaling as \( I (T - T_c)^{-1.2} \) (overdoped state h, \( T_c = 10.6 \text{ K} \)). Inset: \( I = 40 \text{ mA} \), line slope is \(-1.2\).

FIG. 7: Apparent characteristic electrical field from Eq. 1 in a \( \epsilon^{1/2} \) representation (state c, optimally doped). Line is a guide to the eye.

FIG. 8: Characteristic electrical field for the onset of nonlinearity of a Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\) single crystal from [13] in a \( \epsilon^{1/2} \) representation with \( T_c = 78 \text{ K} \). The inset is the same data as presented in [13] and the dotted line accounts for the theoretical \( \epsilon^{3/2} \) behavior.
FIG. 9: Excess conductivity non linearity. Squares: sample c, optimally doped. Circles: Bi$_2$Sr$_{1.7}$La$_{0.3}$CuO$_{6+x}$, slightly underdoped ($T_c/T_c^{max}$=0.98, where $T_c^{max}$ = 30 K). The dotted line is the corrected data for the La substituted sample by a factor $\rho_{pure}/\rho_{substituted}$, so that both samples may be compared for a constant electrical field.